AMERICAN JOURNAL of PHYSICS

A Journal Devoted to the Instructional and Cultural Aspects of Physical Science

VOLUME 20, NUMBER 4 — APRIL, 1952

Velocity-Dependent Potentials for Particles Moving in Given Orbits

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(Received November 5, 1951)

Velocity-dependent potential functions can sometimes be used to determine the field of force that can be applied in order that particles may move in specified paths. In particular, the electromagnetic field vectors **E** and **B** can be determined from such a potential function if the paths on which charged particles move are specified.

The velocity-dependent potential U is related to the kinetic energy T and the Lagrangian function L by the equation

$$U=T-L,$$

where L is an arbitrary solution of the Lagrange equation

$$(d/dt)(\partial L/\partial p')-(\partial L/\partial p)=0,$$

where p = constant represents the orthogonal trajectories of the curves which describe the paths the particles are to follow. From the velocity-dependent potential function U, the field of force can be calculated by the definition

$$Q_p=-(\partial U/\partial p)+(d/dt)(\partial U/\partial p').$$

THE ordinary equations of motion require that one be given the forces which act on a particle. From these given forces the equations of motion are found. The inverse problem is also of interest; namely, the problem of finding a field of force which will cause particles to move in given orbits. Sometimes this field of force can be determined from a velocity-dependent potential function. For example, a velocity-dependent potential function can be used to determine the electromagnetic field in which charged particles must move in order to follow given orbits.

This paper deals with the velocity-dependent potential function and the method used to find the field of force from such a potential function. In particular, the results are applied to examples from the mechanics of charged particles. Using these results, it is possible to determine a field of force if the orbits of particles in the field are given.

In books on mechanics,[1] velocity-dependent potentials are considered from the postulate that the generalized forces Q_j can be calculated from the formula

$$Q_j=(d/dt)(\partial U/\partial q_j')-(\partial U/\partial q_j),$$

where q_j is a generalized coordinate and U is called the generalized or velocity-dependent potential; however, it is of interest to see how to

[1] E. T. Whittaker, *Treatise on the Analytical Dynamics of Particles and Rigid Bodies*, Fourth edition revised (Dover Publications, New York, 1944), p. 44. H. Goldstein, *Classical Mechanics* (Addison-Wesley Press, Inc., Cambridge, Massachusetts, 1950), p. 19.

calculate the velocity-dependent potential if the curves on which the particles move are given.

DETERMINATION OF THE VELOCITY-DEPENDENT POTENTIAL FUNCTION

Suppose it is known that particles have as their orbits certain curves represented by the equation

$$q(u,v)=\text{constant},$$

and that these curves are on a prescribed surface represented by the equations,

$$\begin{aligned} x&=x(u,v),\\ y&=y(u,v),\\ z&=z(u,v). \end{aligned} \tag{1}$$

Let $p(u,v)=\text{constant}$ be the orthogonal trajectories of the curves represented by $q(u,v)=\text{constant}$.

Provided the Jacobian,

$$J=[\partial(q,p)/\partial(u,v)]\neq 0,$$

it is possible to solve the equations $p=\text{constant}$ and $q=\text{constant}$ for u and v in terms of p and q to get

$$\begin{aligned} u&=u(p,q)\\ v&=v(p,q). \end{aligned} \tag{2}$$

Substitution of Eqs. (2) in Eqs. (1) yields

$$x=x[u(p,q),v(p,q)] \tag{3}$$

and similar equations for y and z. Now let $x'=dx/dt$, $y'=dy/dt$, and $z'=dz/dt$. Differentiation of Eq. (3) with respect to t yields

$$x'=(\partial x/\partial p)p'+(\partial x/\partial q)q'. \tag{4}$$

Similar expressions are obtained for y' and z'. Squaring both sides of Eq. (4), we find

$$x'^2=(\partial x/\partial p)^2p'^2+2(\partial x/\partial p)(\partial x/\partial q)p'q' +(\partial x/\partial q)^2q'^2, \tag{5}$$

and similar expressions are obtained for y'^2 and z'^2. Since $p=\text{constant}$ and $q=\text{constant}$ are orthogonal trajectories,

$$\frac{\partial x}{\partial p}\frac{\partial x}{\partial q}+\frac{\partial y}{\partial p}\frac{\partial y}{\partial q}+\frac{\partial z}{\partial p}\frac{\partial z}{\partial q}=0. \tag{6}$$

Therefore, addition of the expressions like Eq. (5) determines the square of the velocity of the particles, but considering Eq. (6) the square of the velocity of the particles reduces to the form,

$$x'^2+y'^2+z'^2=G(p,q)p'^2+H(p,q)q'^2, \tag{7}$$

where $G(p,q)$ and $H(p,q)$ are defined by the equations,

$$\begin{aligned} G(p,q)&=(\partial x/\partial p)^2+(\partial y/\partial p)^2+(\partial z/\partial p)^2\\ H(p,q)&=(\partial x/\partial q)^2+(\partial y/\partial q)^2+(\partial z/\partial q)^2. \end{aligned}$$

Therefore the kinetic energy is given by

$$T=\tfrac{1}{2}m(Gp'^2+Hq'^2).$$

If the particles are to have $q=\text{constant}$ for their orbits, q' must be zero and U must not be a function of q'. Hence, Lagrange's equations of motion can be written from

$$T=\tfrac{1}{2}mGp'^2,\quad U=U(p,q,p'). \tag{8}$$

Lagrange's equations of motion are

$$\begin{aligned} &(m/2)(\partial G/\partial q)p'^2=(\partial U/\partial q)\\ &m\frac{d}{dt}(p'G)-\frac{m}{2}\frac{\partial G}{\partial p}p'^2=-\frac{\partial U}{\partial p}+\frac{d}{dt}\frac{\partial U}{\partial p'}. \end{aligned} \tag{9}$$

The first of Eqs. (9) becomes

$$(\partial/\partial q)(\tfrac{1}{2}mGp'^2)=(\partial U/\partial q), \tag{10}$$

if $[\tfrac{1}{2}mG(\partial p'^2/\partial q)]$ is added to the left-hand side. Since p'^2 and q are independent, this is permissible. Integration of Eq. (10) yields

$$U=\tfrac{1}{2}mGp'^2-L(p,p'). \tag{11}$$

L is some arbitrary function of p and p', but is selected so that U satisfies the second of Eqs. (9). Substitution of the value of U given by Eq. (11) in the last of Eqs. (9) yields the condition

$$(d/dt)(\partial L/\partial p')-(\partial L/\partial p)=0, \tag{12}$$

which must be satisfied by L. Since Eq. (12) is Lagrange's equation, L can be called a Lagrangian. Substitution of Eq. (8) into Eq. (11) yields

$$U=T-L. \tag{13}$$

In the usual form this is written

$$L=T-U.$$

Remembering that $L=L(p,p')$, Eq. (12) can be written

$$p'(\partial^2L/\partial p\partial p')+p''(\partial^2L/\partial p'^2)-(\partial L/\partial p)=0. \tag{14}$$

This second-order partial differential equation has particular solutions which may be combined in any desired form to yield a velocity-dependent potential function U. Some of these solutions are

$$L=\sum_{k=-\infty}^{\infty} A_k p' p^k,$$

$$L=\sum_{l=-\infty}^{\infty} B_l p' p''^l \qquad (15)$$

$$L=C_1(2p''p+p'^2),$$

$$L=C_2(p'^4-12p''p'^2p-12p''^2p^2).$$

The A_k's, B_l's, C_1, and C_2 arear bitrary constants. The last three solutions are given considering p'' as independent of p and p'. It is not necessary that k and l be integers in order that the solutions of Eq. (15) satisfy Eq. (14), but there is no need to consider other than integral values of k and l here. In fact, if the proper form is chosen for the solution, it is only necessary to have the number of arbitrary constants needed to be determined by the boundary conditions given.

CALCULATION OF THE FIELD OF FORCE

Calculation of the field $\mathbf{F}$ from a velocity-dependent potential U is accomplished by considering the generalized force derived from U,

$$-(\partial U/\partial p)+(d/dt)(\partial U/\partial p')=Q_p. \qquad (16)$$

Since Eq. (16) holds for a generalized coordinate p, it holds for the rectangular Cartesian coordinates x, y, and z. Therefore, the field components are given by

$$F_x=-(\partial U/\partial x)+(d/dt)(\partial U/\partial x'),$$
$$F_y=-(\partial U/\partial y)+(d/dt)(\partial U/\partial y'), \qquad (17)$$
$$F_z=-(\partial U/\partial z)+(d/dt)(\partial U/\partial z').$$

These equations can be written in vector form if ∇_v is defined by the equation,

$$\nabla_v U=[(\partial\mathbf{i}/\partial x')(\partial\mathbf{j}/\partial y')(\partial\mathbf{k}/\partial z')]U,$$

where $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ are unit vectors in the x, y, and z directions, respectively. Equations (17) can be written as

$$\mathbf{F}=-\nabla U+(d/dt)\nabla_v U. \qquad (18)$$

Here ∇U denotes the usual operation of taking the gradient of U.

ELECTROMAGNETIC FIELDS

An electromagnetic field can be derived from a velocity-dependent potential function. An electromagnetic field is detected by placing a charge q in the field. The charge experiences a force given by

$$\mathbf{F}=q[\mathbf{E}+\mathbf{v}\times\mathbf{B}], \qquad (19)$$

where $\mathbf{B}$ is the magnetic induction, $\mathbf{E}$ is the electric intensity, and $\mathbf{v}$ is the velocity of the charged particle. Maxwell's equations describe the fields $\mathbf{E}$ and $\mathbf{B}$

$$\text{(I)}\quad \nabla\times\mathbf{E}=-(\partial\mathbf{B}/\partial t) \qquad \text{(III)}\quad \nabla\cdot\mathbf{D}=\rho$$
$$\text{(II)}\quad \nabla\times\mathbf{H}=\mathbf{J}+(\partial\mathbf{D}/\partial t) \qquad \text{(IV)}\quad \nabla\cdot\mathbf{B}=0.$$

By virtue of Eq. (IV), $\mathbf{B}$ is a solenoidal vector and can be expressed as the curl of some other vector $\mathbf{A}$. Hence,

$$\nabla\times\mathbf{A}=\mathbf{B}, \qquad (20)$$

and Eq. (I) becomes

$$\nabla\times[\mathbf{E}+(\partial\mathbf{A}/\partial t)]=0.$$

Thus the field $[\mathbf{E}+(\partial\mathbf{A}/\partial t)]$ is irrotational and may be derived from the negative of the gradient of some scalar function ϕ, or

$$\mathbf{E}=-\nabla\phi-(\partial\mathbf{A}/\partial t). \qquad (21)$$

Substitution of Eq. (21) in Eq. (19) yields

$$\mathbf{F}=q[-\nabla\phi-(\partial\mathbf{A}/\partial t)+\mathbf{v}\times\nabla\times\mathbf{A}].$$

Just the x component of this force can be written as

$$F_x=q[-(\partial\phi/\partial x)-(dA_x/dt)(\partial/\partial x)(\mathbf{A}\cdot\mathbf{v})], \qquad (22)$$

for

$$F_x=q\left[-\frac{\partial\varphi}{\partial x}-\frac{\partial A_x}{\partial t}+y'\left(\frac{\partial A_y}{\partial x}-\frac{\partial A_x}{\partial y}\right)-z'\left(\frac{\partial A_x}{\partial z}-\frac{\partial A_z}{\partial x}\right)\right].$$

If $q(\partial A_x/\partial x)x'$ is added to and subtracted from the right-hand side of this equation, it can be written in the form of Eq. (22). A similar argument can be given for the y and z components of the force $\mathbf{F}$. Therefore, the components of the

force on a charged particle are given by

$$F_x=q[-(\partial\phi/\partial x)+(\partial/\partial x)(\mathbf{A}\cdot\mathbf{v})-(d/dt)A_x],$$
$$F_y=q[-(\partial\phi/\partial y)+(\partial/\partial y)(\mathbf{A}\cdot\mathbf{v})-(d/dt)A_y], \quad (23)$$
$$F_z=q[-(\partial\phi/\partial z)+(\partial/\partial z)(\mathbf{A}\cdot\mathbf{v})-(d/dt)A_z].$$

Comparison of Eqs. (23) with Eqs. (17) shows that

$$(\partial U/\partial x')=-A_xq,\quad (\partial U/\partial y')=-A_yq,\quad (\partial U/\partial z')=-A_zq, \quad (24)$$

and

$$\phi=(U/q)-(\mathbf{A}\cdot\mathbf{v}). \quad (25)$$

Then $\mathbf{E}$ and $\mathbf{B}$ are computed from relations (20) and (21). This shows that the electric and magnetic fields can be determined by the use of a velocity-dependent potential function if the orbits of particles in the field are known. The field will not be unique but will contain arbitrary constants which can be used to specify special conditions that the motion of the particles may follow. It is possible that ϕ may be a function of the velocities x', y', and z'. If this is the case, it is assumed that x', y', and z' are given as functions of time so that ϕ may be considered a function of the space coordinates and time.

EXAMPLE

As an example, consider the problem of determining the electromagnetic field in free space which would cause particles moving on the surface represented by

$$x=a\cos u,\quad y=a\sin u,\quad z=v,\quad \text{when}\quad v\leq 0, \quad (26)$$

and by

$$x=ae^{-bv^2}\cos u,\quad y=ae^{-bv^2}\sin u,\quad z=v,\quad \text{when}\quad v>0, \quad (27)$$

to move on curves represented by

$$q=u=\text{constant}. \quad (28)$$

Further, the velocity component z' is specified as a function of time

$$z'=z_0''t, \quad (29)$$

where z_0'' is a constant acceleration component. The particles are assumed to start from rest. The orthogonal trajectories of the curves (28) are represented by

$$p=v=\text{constant}. \quad (30)$$

Solving for u and v from Eqs. (28) and (30) and substituting the results in Eqs. (26) and (27), we find

$$x=a\cos q,\quad y=a\sin q,\quad z=p, \quad (31)$$

and

$$x=ae^{-bp^2}\cos q,\quad y=ae^{-bp^2}\sin q,\quad z=p. \quad (32)$$

From Eqs. (31) and (32) it is found that the kinetic energy is given by the expressions,

$$T=\tfrac{1}{2}mp'^2, \quad (33)$$

when $z\leq 0$, and

$$T=\tfrac{1}{2}m(1+4a^2b^2p^2e^{-2bp^2})p'^2, \quad (34)$$

when $z>0$. The derivative q' has been set equal to zero in the expressions for the kinetic energy of the particles since the particles are to move on curves $q=\text{constant}$.

Comparing Eqs. (33) and (34) with the first of Eqs. (8) we see that

$$G(p,q)=1,$$

when $z\leq 0$, and that

$$G(p,q)=1+4a^2b^2p^2e^{-2bp^2},$$

when $z>0$. Therefore Eq. (11) yields the velocity-dependent potential function

$$U=\tfrac{1}{2}mp'^2-L(p,p') \quad (35)$$

when $z\leq 0$, and

$$U=\tfrac{1}{2}m(1+4a^2b^2p^2e^{-2bp^2})p'^2-L(p,p'), \quad (36)$$

when $z>0$. From Eqs. (15) a solution for L is chosen as

$$L=\sum_{k=-\infty}^{\infty}A_kp'p^k.$$

Noting from Eqs. (31) and (32) that

$$p=z,\quad x^2+y^2=a^2e^{-2bp^2},$$

the expressions (35) and (36) can now be written as

$$U=\tfrac{1}{2}mz'^2-\sum_{k=-\infty}^{\infty}A_kz'z^k, \quad (37)$$

when $z \leq 0$, and

$$U = \tfrac{1}{2}m[1+4b^2z^2(x^2+y^2)]z'^2 - \sum_{k=-\infty}^{\infty} A_k z' z^k, \quad (38)$$

when $z>0$. The components of the vector potential are given by Eqs. (24) as

$$A_x = A_y = 0, \quad A_z = -\frac{m}{q}z' + \frac{1}{q}\sum_{k=-\infty}^{\infty} A_k z^k, \quad (39)$$

when $z \leq 0$ and

$$A_x = A_y = 0,$$

$$A_z = -\frac{m}{q}[1+4b^2z^2(x^2+y^2)]z' + \frac{1}{q}\sum_{k=-\infty}^{\infty} A_k z^k, \quad (40)$$

when $z>0$. The scalar potential function is given by Eq. (25) as

$$\varphi = \frac{3}{2q}mz'^2 - \frac{2}{q}\sum_{k=-\infty}^{\infty} A_k z' z^k, \quad (41)$$

when $z \leq 0$, and as

$$\varphi = \frac{3}{2q}m[1+4b^2z^2(x^2+y^2)]z' - \frac{2}{q}\sum_{k=-\infty}^{\infty} A_k z' z^k, \quad (42)$$

when $z>0$. From these potential functions, **E** and **B** can be determined by use of the relations (20) and (21). On substituting for z' from Eq. (29), the components of **E** and **B** are

$$E_x = E_y = 0, \quad E_z = -\frac{2}{q}\sum_{k=-\infty}^{\infty} kA_k z_0'' t z^{k-1} + \frac{mz_0''}{q},$$

$$B_x = B_y = B_z = 0,$$

when $z \leq 0$; and when $z>0$ the components of **E** and **B** are given by

$$E_x = (12mb^2z_0''z^2xt)/q, \quad E_y = (12mb^2z_0''z^2yt)/q,$$

$$E_z = \frac{12mb^2zz_0''t}{q}(x^2+y^2)$$

$$-\frac{2}{q}\sum_{k=-\infty}^{\infty} kA_k z_0'' z^{k-1} t - \frac{mz_0''}{q}[1+4b^2z^2(x^2+y^2)];$$

$$B_x = -(m/q)8b^2z^2yz_0''t, \quad B_y = (m/q)8b^2z^2xz_0''t,$$

$$B_z = 0.$$

The A_k's are as yet arbitrary. They can therefore be used in selecting convenient values for the z component of the electric field intensity for a given position and the corresponding time.

CONCLUSION

Given the paths in free space on which it is desired that particles move, it is possible to determine the field of force which can be applied. The solution may involve the velocity components of the particles. In such cases the velocity components must also be specified in order that the field of force be determined from a velocity-dependent potential function.

. . . we can see in this example . . . how in palpable bodies, composed of many atoms, individuality arises out of the structure of their composition, out of shape or form, or organization, as we might call it in other cases. The identity of the material, if there is any, plays a subordinate role. You may see this particularly well in cases when you speak of "sameness" though the material has definitely changed. A man returns after twenty years of absence to the cottage where he spent his childhood. He is profoundly moved by finding the place unchanged. The same little stream flows through the same meadows, with the cornflowers and poppies and willow trees he knew so well, the white-and-brown cows and the ducks on the pond, as before, and the collie dog coming forth with a friendly bark and wagging his tail to him. And so on. The shape and the organization of the whole place have remained the same, in spite of the entire "change of material" in many of the items mentioned, including, by the way, our traveller's own bodily self! Indeed, the body he wore as a child has in the most literal sense "gone with the wind." Gone, and yet not gone. For, if I am allowed to continue my novelistic snapshot, our traveller will now settle down, marry, and have a small son, who is the very image of his father as old photographs show him at the same tender age.
—E. Schrödinger, Science and Humanism (1951).

Undergraduate Origins of American Physicists

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(Received November 21, 1951)

The number of physicists listed in the 1949 edition of *American Men of Science* who have received their undergraduate degrees from various colleges and universities has been tabulated. For each institution the ratio of the number of listings in physics to the total enrollment has been computed. A comparison of these data with those available from previous editions suggests that universities having major graduate programs in physics are becoming increasingly important in the training of undergraduates.

ONE of the prime requirements in the development of physicists is a steady stream of able college graduates, well grounded in the basic principles of physics and mathematics. Certain institutions have made very special contributions in this field of nursing young physicists to the point at which they can be transplanted to graduate school for further growth. One estimate of the relative importance of an institution in training undergraduate physicists can be obtained by counting the number of physicists listed in *American Men of Science* who have received their undergraduate training at the particular institution in question. Blackwood[1] published a paper on the undergraduate origins of American physicists listed in the 1938 edition of *American Men of Science;* the present paper makes a similar report on the men listed as physicists in the 1949 edition.

Slightly over 4200 persons are listed in the field of physics in the most recent edition of *American Men of Science.* This number represents some 58 percent of the number of members of the American Physical Society. The sole criterion used in assigning a man to the field of physics was that the word *physics* appear in his field of specialization. Thus, men in astrophysics and biophysics are included, but those listed in electronics, physical chemistry, and the various branches of engineering are not. In this arbitrary classification there are undoubtedly some physicists whose records were not considered and others included as physicists whose principle training was in a related field. However, the number of such misassignments should be small and it may be hoped that this summary gives a reasonably reliable picture.

Table I lists the number of physicists in *American Men of Science* who received undergraduate degrees from each of the 73 institutions which contributed ten or more names. The data of this table are in excellent agreement with the findings of Trytten[2] who has reported the undergraduate origins of all persons receiving Ph.D.'s in physics in the decade 1936–1945.

A glance at Table I reveals that the universities which have great graduate centers in physics are also very important contributors of undergraduates. Indeed, of the first 32 institutions listed in Table I only three had awarded fewer than 14 Ph.D.'s in physics to persons in *American Men of Science;* of the first fifty institutions only four had awarded no physics Ph.D.'s at all. This is in sharp contrast to Blackwood's findings in connection with the 1938 edition in which seven of the leading thirty institutions were colleges which did not offer the doctoral degree. The detailed report by White[3] on enrollments and degrees granted to physics majors during the academic year 1949–1950 shows that of the twenty institutions having the largest number of graduate students eleven are also among the twenty institutions having the largest number of undergraduate physics majors. It appears that universities with major graduate schools are becoming relatively more important in the training of undergraduate physicists.

Clearly, in the straightforward counting of graduates large institutions have a tremendous advantage over small ones. A "productivity

[1] O. Blackwood, Am. J. Phys. **12**, 149 (1944).

[2] M. H. Trytten, Am. J. Phys. **15**, 330 (1947).

[3] M. W. White, Am. J. Phys. **19**, 27 (1951).

Table I. Number of persons listed as physicists in the 1949 edition of *American Men of Science* who received undergraduate degrees from the institutions listed.

Rank	Number	Institution	Location
1	139	Massachusetts Inst. of Technology	Mass.
2	92	Harvard	Mass.
3	89	California (Berkeley)	Calif.
4	88	Chicago	Ill.
5	84	Michigan	Mich.
6	78	Wisconsin	Wisc.
7	76	Cornell	N. Y.
8	66	City College of New York	N. Y.
9	65	California Inst. of Technology	Calif.
10	62	Columbia	N. Y.
11	58	Indiana	Ind.
12	55	Yale	Conn.
13	48	Toronto	Ont.
14	46	Princeton	N. J.
15	42	Illinois	Ill.
16		Minnesota	Minn.
17	39	Case Inst. of Technology	Ohio
18	18	Ohio State	Ohio
19	36	Johns Hopkins	Md.
20		Carnegie Inst. of Technology	Pa.
21	33	George Washington	D. C.
22		Texas	Tex.
23	30	Virginia	Va.
24		Iowa	Iowa
25	28	Pennsylvania State	Pa.
26		Rochester	N. Y.
27	27	Colorado	Colo.
28		Stanford	Calif.
29		Cincinnati	Ohio
30	26	McGill	Que.
31		Northwestern	Ill.
32	25	Nebraska	Neb.
33	24	British Columbia	B. C.
34		California (Los Angeles)	Calif.
35	23	Iowa State	Iowa
36		Missouri	Mo.
37		Dartmouth	N. H.
38	22	Pittsburgh	Pa.
39		Saskatchewan	Sask.
40		Kansas	Kan.
41	20	Purdue	Ind.
42		Univ. of Washington	Wash.
43		Kentucky	Ky.
44	19	Oklahoma	Okla.
45		Union	N. Y.
46		Lehigh	Pa.
47	18	Rice Inst.	Tex.
48		Rutgers	N. J.
49	17	New York	N. Y.
50	16	Brooklyn	N. Y.
51		Brown	R. I.
52		Miami	Ohio
53	14	Oregon State	Ore.
54		Swarthmore	Pa.
55		Kalamazoo	Mich.
56	13	Rensselaer Polytechnic Inst.	N. Y.
57		West Virginia	W. Va.
58		Dalhousie	Nova Scotia
59	12	Knox	Ill.
60		Queens	N. Y.
61		Boston Univ.	Mass.
62		Clark	Mass.
63		Michigan State	Mich.
64	11	Oregon	Ore.
65		Washington (St. Louis)	Mo.
66		Montana State	Mont.
67		St. Olaf	Minn.
68		Franklin and Marshall	Pa.
69		North Carolina	N. C.
70	10	Oberlin	Ohio
71		Ohio Wesleyan	Ohio
72		St. Louis	Mo.
73		Syracuse	N. Y.

index," convenient for comparing the relative merits of institutions with greatly different sizes, may be obtained by dividing the number of successful graduates by the enrollment. Such an index has been computed for the more important institutions by dividing the number of graduates listed as physicists in *American Men of Science* by the total enrollment in thousands for the academic year 1935–1936 as published in the *1937 World Almanac and Book of Facts*. This index has the property of unduly favoring noncoeducational institutions, and other institutions which have an unusually large fraction of the total enrollment in an undergraduate scientific program.

Table II lists the productivity indices for the 65 institutions for which the index exceeds 10.0. As might be expected, several of the best schools of technology rank high. Also prominent are a number of small liberal arts colleges. The excellent showing of these schools is doubtless associated with the inspirational teaching of one or two extraordinary men. Several of these exceptional teachers have been honored with the Oersted Medal. The Oersted Medalist for 1950, the late Professor John W. Hornbeck, is primarily responsible for the high standing of Kalamazoo College.

Knapp and Goodrich[4] have computed an index of production of scientists in general. Although many of the institutions which have a high index of production in their report also appear prominently in Table II, there are certain significant differences which can be noted. In particular, Knapp and Goodrich find that southern and far eastern institutions are relatively rare in the list of the 50 institutions which lead in the production of scientists, while Table II shows

[4] R. H. Knapp and H. B. Goodrich, Science 113, 543 (1951).

TABLE II. Number of graduates listed as physicists in the 1949 edition of *American Men of Science* per thousand students enrolled in 1935–1936 according to the *1937 World Almanac and Book of Facts.*

Rank	Number per thousand	Institution	Location	Rank	Number per thousand	Institution	Location
1	78.3	California Inst. of Technology	Calif.	33	13.6	Furman	S. C.
2	53.9	Massachusetts Inst. of Technology	Mass.	34	13.4	Allegheny	Pa.
				35	13.3	Dickinson	Pa.
3	47.6	Case Inst. of Technology	Ohio	36	13.3	Knox	Ill.
4	41.8	Kalamazoo	Mich.	37	13.3	Reed	Ore.
5	39.7	Clark	Mass.	38	13.3	Kenyon	Ohio
6	30.5	Milton	Wisc.	39	12.9	Cornell	N. Y.
7	27.4	Friends	Kan.	40	12.9	Franklin and Marshall	Pa.
8	24.6	Johns Hopkins	Md.	41	12.8	Juniata	Pa.
9	23.8	Union	N. Y.	42	12.8	Virginia	Va.
10	22.6	Knox	Ill.	43	12.7	British Columbia	B. C.
11	22.5	North Central	Ill.	44	12.5	Lehigh	Pa.
12	21.6	Swarthmore	Pa.	45	12.4	Bates	Me.
13	21.4	Haverford	Pa.	46	12.3	Willamette	Ore.
14	20.6	Nebraska Wesleyan	Neb.	47	12.2	Phillips	Okla.
15	20.2	Ripon	Wisc.	48	11.9	Wesleyan	Conn.
16	20.1	Columbia	N. Y.	49	11.7	Harvard	Mass.
17	20.0	Hobart	N. Y.	50	11.6	Bowdoin	Me.
18	19.8	William Jewell	Mo.	51	11.6	Colorado School of Mines	Colo.
19	18.4	Wabash	Ind.	52	11.5	Muhlenberg	Pa.
20	17.9	Princeton	N. J.	53	11.3	Albion	Mich.
21	15.8	Olivet	Mich.	54	11.3	Hiram	Ohio
22	15.6	Saskatchewan	Sask.	55	11.3	Rensselaer Polytechnic Inst.	N. Y.
23	15.5	Park	Mo.	56	11.2	Indiana	Ind.
24	15.0	Earlham	Ind.	57	11.1	Lake Forest	Ill.
25	14.9	Dalhousie	Nova Scotia	58	11.1	Morningside	Iowa
26	14.7	Rochester	N. Y.	59	11.0	Illinois Wesleyan	Ill.
27	14.6	Carnegie Inst. of Technology	Pa.	60	10.9	St. Olaf	Minn.
28	14.5	Emporia	Kan.	61	10.9	Whitman	Wash.
29	14.0	McPherson	Kan.	62	10.8	Yale	Conn.
30	13.8	Worcester Polytechnic Inst.	Mass.	63	10.7	St. Lawrence	N. Y.
31	13.7	Rice Inst.	Tex.	64	10.3	McGill	Que.
32	13.6	Dayton	Ohio	65	10.0	Muskingum	Ohio

much better representation of New England and Middle Atlantic institutions.

A comparison of the productivity indices in Table II with a related index computed by Blackwood[1] for the 1938 edition of *American Men of Science* again suggests that the universities which have important graduate schools are playing a more vital role in training undergraduates. Blackwood's productivity index was calculated by dividing the number of listed graduates of each institution by the number of male students enrolled in 1925–1926. In so far as information permitted, liberal arts and pre-engineering students were included, but preprofessional students of agriculture, architecture, business, education, and the like were not. Since the two productivity indices were computed on a somewhat different basis, the order in which institutions place would be somewhat different in the two cases. Probably the present index treats large universities less well than did Blackwood's since it is these universities rather than the small colleges which have preprofessional students in the fields that Blackwood did not include in his calculations. Of Blackwood's list of the 29 most productive institutions only four are major graduate centers, while of the corresponding top 29 of Table II, six are so regarded. Not only are more important graduate centers represented in the list, but their positions are considerably higher.

The limited data presented here suggest that there may be a trend away from the familiar pattern in which small liberal arts colleges serve as the prime source of graduate students toward one in which the major universities provide a larger share of the undergraduate training of physicists.

Experimental Study of Sliding Friction

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(Received September 17, 1951)

This article constitutes a report on an independent experimental study of the variation of the coefficient of sliding friction under very simple laboratory conditions. It is based on nonaccelerated motion involving sliding friction at speeds from 0.25 to 20 cm/sec. The experimental set-up consists of a weighted block resting upon a horizontal surface, subjected to the pull of various weights suspended over a pulley and attached to the block by means of a cord.

For all materials tested, including wood, leather, glass, and various metals, it is found that (1) the coefficient of sliding friction varies as the logarithm of the speed of the moving surface, increasing directly as the logarithm of the speed increases, (2) in the case of wood surfaces in contact, the coefficient of sliding friction decreases as the normal force increases, (3) in the case of both wood and leather surfaces in contact with other surfaces, the coefficient decreases with repeated performance, and (4) in the case of metal surfaces in contact, the coefficient of friction increases with repeated performance, unless the product of the abrasive action of friction is removed for each performance. The removal of the loose metallic particles causes a decreased coefficient of friction.

CONSIDER a body of weight W_1 resting on a horizontal plane and connected by a cord over a pulley to a suspended weight W_2 (Fig. 1). Assume the coefficient of sliding friction between the body and the plane to be a constant μ, and the pulley to be weightless and frictionless. The resulting acceleration a is represented by the formula[1]

$$a=[(W_2-\mu W_1)/(W_1+W_2)]g,$$

where g is the acceleration of gravity.

This formula is based upon the assumption of a constant coefficient of sliding friction, an assumption which has been found to be not in accordance with observed data. The coefficient of sliding friction increases in proportion to the logarithm of the speed, as will be shown below. Consequently, the unbalanced force $W_2-\mu W_1$ acting upon (W_1+W_2) decreases with increasing speed until the speed becomes uniform and $W_2=\mu W_1$. The starting acceleration of the system continues to decrease along with the decreasing unbalanced force and so the acceleration is not constant at any time. Where the unbalanced force is initially small, the speeds produced are also small and the state of uniform speed is reached almost immediately. The ratio W_2/W_1 thus measures the coefficient of sliding friction for a given speed.

Although the variation of the coefficient of sliding friction with the logarithm of the speed represents an independent deduction from recent extensive measurements, an examination of the literature on friction shows that this same law of variation was also stated by the celebrated physicist, Charles A. Coulomb,[2] in 1785 in connection with some of his experiments on sliding friction between wood and metal surfaces. Coulomb's generalization "friction increases in arithmetical progression as the speed increases in geometrical progression" is identical in meaning with the statement that the coefficient of friction varies directly as the logarithm of the speed. However, in drawing up a final statement on the sum of his experimental work on friction, Coulomb[3] merely concludes "It is not always that, with wood sliding without lubrication on wood and with metals sliding on metals, the

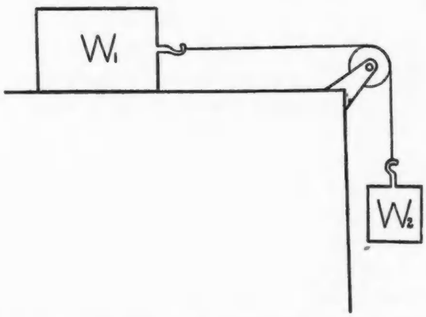

FIG. 1. Diagram of simple equipment used to measure coefficients of sliding friction under normal operating conditions.

[1] E. Hausmann and E. P. Slack, *Physics* (D. Van Nostrand Company, Inc., New York, 1944), U. S. Naval Academy revised edition, p. 80.

[2] Frederic Palmer, Am. J. Phys. **17**, 185 (1949).

[3] Reference 2, p. 185.

speed has only slight influence upon the friction; but here the friction increases very sensibly in proportion as the speed is increased."

The more elaborate tests of Morin[4] reported in 1831 led Morin to abandon Coulomb's theory that "the friction of metals on wood ought to increase with speed." It is to be noted that in checking Coulomb's idea of the possible increase of friction with speed, Morin[4] extended the range of his measurements to include greater velocities up to "3.5 meters per second or more." Measurements with these higher velocities failed to show what measurements with decreased velocities would have shown. The variation of the coefficient, depending as it does upon the logarithm of the speed, is very marked at slow speeds of a few cm/sec, but is scarcely perceptible at speeds of the order of 3 meters/sec. This point is well illustrated by data contained in the present article.

COEFFICIENT OF FRICTION NOT A CONSTANT

The normal occurrence of friction phenomena is under uncontrolled conditions, and the performance of friction under such conditions indoors constitutes the problem here presented. Data have been secured for a variety of combinations of dry or unlubricated pairs of surfaces, displaying the nature of the variation of the coefficient of friction with speed and other factors. The data in all of the separate experiments reported bear out the generalization that the coefficient of friction varies as the logarithm of the speed. The range of speed covered is from 0.25 to 20 cm/sec. It is believed, however, that the generalization applies to a much wider range of speeds than represented by the data of this study. A very slow speed of a wooden block over a steel surface was easily attained in one instance not represented by data reported here. At least three hours were required for the block to move a distance of 100 centimeters over the steel surface. This motion corresponds to a speed of less than 0.01 cm/sec. The speeds in every case considered were the nonaccelerated limiting speeds acquired almost immediately after the motion was started.

In the case of wood surfaces, evidence is presented from two different approaches displaying the decrease of the coefficient of friction with increase of normal force. In the first instance data were chosen corresponding to the same speed throughout. That is to say, the factor of speed was kept very nearly constant, the two variable factors being the coefficient of friction and the normal force. In the second instance three separate sets of readings corresponding to three different fixed normal forces were made throughout the range of variation of speed. The data secured in both instances consistently reflect the law of decreasing coefficient of friction with increasing normal force in the case of wood surfaces.

The decrease of the coefficient of friction with repeated performance, in the case where one of the two surfaces is of fibrous composition, as is true of wood or leather, is illustrated by data pertaining to the friction between leather and steel.

The variation of the coefficient of friction where both surfaces are of metal shows the very same law of relationship with speed as obtains where one or both of the surfaces is of fibrous composition. However, metal surfaces were found to be subject to more variable factors than were fibrous surfaces. Different curves, each consistent for a given time of observation, were found on different days. Atmospheric conditions possibly had something to do with these differences. It is definitely established in these experiments, however, that the degree of cleansing of the metal surfaces has much to do with the variation of the coefficient. In these separate groups of experiments, the friction of lead on steel, of brass on steel, and of aluminum on steel, the effect of cleansing was found to be the same for all. The mere wiping of the surfaces between runs with clean cotton cloths was found necessary to secure consistent results. The cleansing of the surfaces between runs with common dry-cleaning agents has a marked effect on the results. It appears definitely established that an abrasive action between metal surfaces takes place when one metal moves over another, and the tiny loosened particles of metal serve to increase the friction as they accumulate. The repeated removal of this abraded material by a dry-cleaning agent causes a decreased coefficient of friction.

[4] Reference 2, p. 186.

EXPERIMENTAL PROCEDURE

The experimental set-up for all of the data secured is so simple as to require no special diagram. A weighted block of one material is made to slide a short distance of approximately 100 centimeters over a horizontal board or sheet of another material. A light-weight cord attached to the block passes over a small pulley at the end of the board and is attached to a weight holder on which various weights are placed. Since the coefficient of static friction is in every case greater than that of sliding friction, the block could be rested on the surface until a slight jar started it in motion. The starting jar has the effect of giving the block a slight lift from the surface, producing first an acceleration which shortly reduces to nonaccelerated motion or a limiting speed.

Some of the first data studied were measures taken of electrically recorded speeds on sensitized paper tape spread along the horizontal path. As the work got under way further, it was found that with a very slight jar used to start the motion there was no practical difference in the calculations secured by ignoring the slight acceleration effects at starting. Stop watch recordings of times required to travel given distances were secured much more quickly and easily than tape recordings and were sufficiently accurate to obtain the generalizations of this study.

It should be noted that the measurement of the force of friction presents no problem at all when, as in the case of the method employed here, this force is an arbitrarily chosen value. The normal force, of course, is also chosen arbitrarily, and the ratio of the former to the latter being precisely determined as the friction coefficient, is not subject to any significant probable error. Fluctuations in speed due to the ever-present irregularities of surface conditions are reducible in magnitude by patient working over of the surfaces. Such minor fluctuations as remain can be logically attributed to local surface irregularities, provided the times of repeated runs for the same friction force prove consistently to be nearly the same.

Fish-line cord of a total mass of three-tenths of a gram per meter length proved suitably strong for most of the measurements. The increments of weight added to the pulling weight were mostly 20 grams, but ranged from 5 to 50 grams. Since the increments of cord over the pulley were the same in all of the compared runs and represented relatively small variable increments of pull, corrections due to this factor were neglected. The small brass pulley used had little moment of inertia, and since the motion was nonaccelerated, none of the pulling force was used in producing angular momentum of the pulley. The friction of the pulley was measured for different loads and taken into account in the calculations. This correction factor proved to amount to approximately three percent of the load over the pulley.

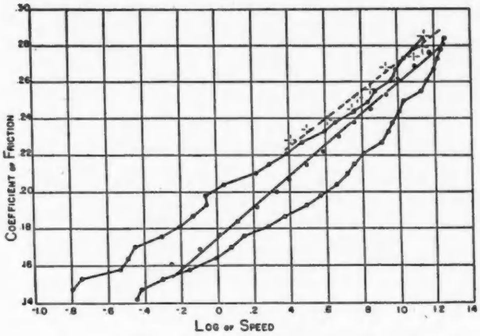

FIG. 2. Comparison of coefficients of friction for poplar on various surfaces.

A considerable amount of preliminary experimentation preceded the actual gathering of data in each of the problems attacked. Different types of surfaces required different kinds of preparation. For example, the wood surfaces, after sandpapering and wiping, were put through several sets of runs until the surfaces acquired what might be termed a "seasoned" condition. Metal surfaces were rendered devoid of foreign material such as lubricants or cleansing agents so far as possible. Some of the cleansing agents used in the preparation of metal surfaces, such as perchloroethylene, were wholly volatile. Clean cotton cloths were used in connection with these agents to remove by wiping all foreign material that could be removed by such a process.

DISCUSSION OF RESULTS

The solid straight line in Fig. 2 is a plot of the coefficient of friction *versus* the logarithm of the speed for data secured with a poplar block mov-

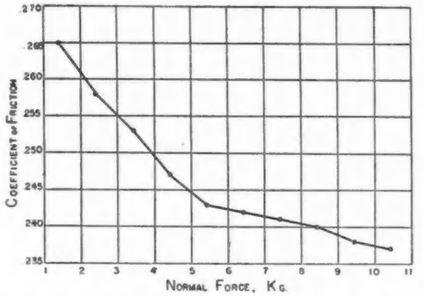

FIG. 3. Variation of coefficient of friction with normal force. Poplar block on poplar board. Speed constant at 13.0 cm/sec.

ing over a poplar board. Increments of 50 grams of pull were employed, starting with a gross pull of 1050 grams. The moving poplar block, with a "contact surface" area of some 200 cm² weighed together with loaded weights a total of 6367 grams. The so-called area of surface contact is a misleading figure. Actually the contact surface is but a small fraction of the measured area; it varies from place to place and from time to time due to the ever-present unevennesses inherent in any prepared level surfaces. The variations in actual area of surface of contact were not found to be such as to prevent the obtaining of significant measurements leading to significant generalizations.

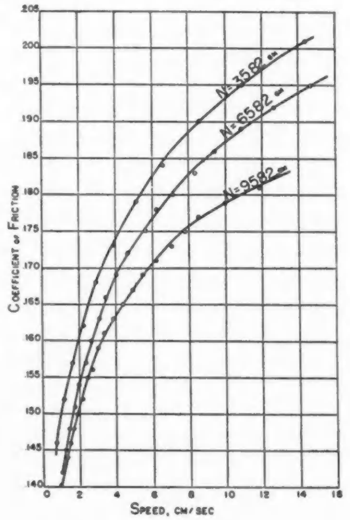

FIG. 4. Variation of coefficient of friction with speed and with normal force. Pine block on walnut board.

Each plotted point of the solid straight line represents the average of merely two measures of electrical recordings. Considering the small amount of data used in making this chart, the consistently linear character of the plot of coefficient of friction *versus* logarithm of the speed stands out as a good illustration of the law of variation of the coefficient. It will be noted in the discussion that follows that the separate points plotted in various charts represent means of several observations, up to 30 or more separate measures for each point, in some instances.

One of the two runs referred to above was made by increasing the pulling force in steps of

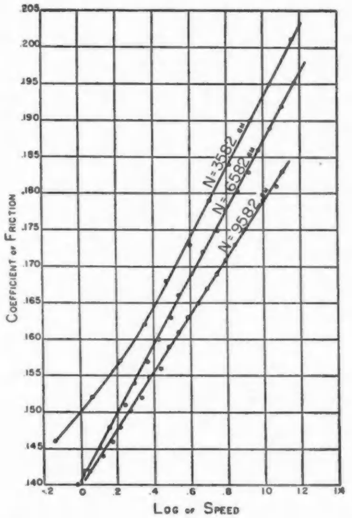

FIG. 5. Variation of coefficient of friction with logarithm of speed and with normal force. Pine block on walnut board.

50 grams, and the other by decreasing the pulling force in steps of 50 grams back to the original load of 1050 grams. This procedure tended to offset the effect of variation of friction with repeated performance. There was no question as to the nonaccelerated character of the motion. Countless hundreds of tests made by stop watch as well as by electrical recording on sensitive paper have shown that the motion of the sliding surface, within the range of speed studied, is a nonaccelerated motion. Slight fluctuations in the speed due to ever-present irregularities in the surfaces are, of course, obtained. But the signicant fact is that in this speed range there is no

net acceleration of the moving surface, which definitely means that friction "absorbs" all of the pull. This, of course, means that the coefficient of friction increases with the speed.

In the measurements of this particular experiment the range of variation of the coefficient is large, its value being 0.161 for a speed of 0.56 cm/sec and 0.276 for a speed of 14.84 cm/sec, an increase of 70 percent.

The short dotted line on Fig. 2 displays the variation of the coefficient of friction with the logarithm of the speed for a poplar block moving on a surface of plate glass. Each plotted point marked by a cross represents the mean of from 3 to 6 separate observations.

The two broken lines on Fig. 2 display two separate plots for the same pair of surfaces, and apply to the motion of a poplar block over a steel surface. Each plotted point represents the mean of from 2 to 4 separate observations. The upper broken line reflects the variation of the coefficient before a special base metal burnisher was applied, and the lower broken line gives the results of observations taken immediately after the burnishing and cleansing. The effect of the special burnishing is seen to be a general reduction in the friction.

It is to be noted that the graphs in Fig. 2 are made from comparatively few observations and that they are somewhat exploratory in character. The same poplar block was used for the four sets of experimental readings. There appears to be a tendency for the coefficients to become nearly equal for the different surfaces at the highest speeds used.

Figure 3 displays the variation of the coefficient of friction with the normal force. The same poplar block and board used in furnishing the data for the solid straight line of Fig. 2 were employed in this study. Kilogram weights were added to the block to furnish the additional normal forces. Each of the ten plotted points represents the mean of 33 runs made on seven different days. The pulley weights required for producing an arbitrary speed of 13.0 cm/sec required careful adjusting, but in all cases the average speed was kept very close to this figure. The graph shows a definite decrease in the coefficient with an increase in the normal force. The large number of observations taken gives validity to this finding. In this, as in all the separate experiments carried out and reported here, correction was made for the pulley friction.

TABLE I. Variation of coefficient of friction with normal force. Pine block on walnut board.

Gross friction force (grams)	Speed (centimeters per second)	Adjusted coefficient of friction	Logarithm of speed
1380	0.96	0.140	−0.018
1400	1.12	0.142	+0.049
1420	1.32	0.144	+0.121
1440	1.49	0.146	0.173
1460	1.64	0.148	0.215
1480	1.88	0.150	0.274
1500	2.17	0.152	0.336
1520	2.41	0.155	0.382
1540	2.73	0.156	0.436
1560	3.02	0.159	0.480
1580	3.42	0.161	0.534
1600	3.87	0.163	0.588
1620	4.41	0.165	0.644
1640	4.97	0.167	0.696
1660	5.50	0.169	0.740
1680	6.21	0.171	0.793
1700	7.07	0.173	0.849
1720	7.81	0.175	0.893
1740	8.54	0.177	0.931
1760	10.00	0.179	1.000
1780	11.88	0.181	1.075
1800	12.81	0.183	1.108

Normal force = 9582 grams
Length of path = 100 centimeters.
22 points; each point is the mean of 8 observations.

Figures 4 and 5 depict the variation of the coefficient when using a block with pine runners moving over a walnut board. Three different normal forces of 3582, 6582, and 9582 grams weight were used for the three sets of runs.

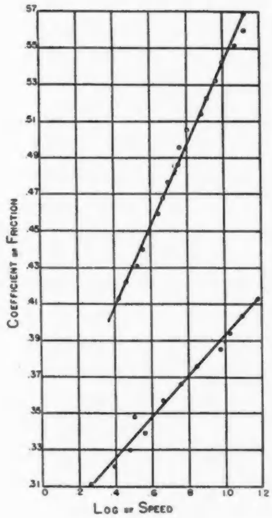

FIG. 6. Reduction in friction by repeated performance. Between first and sixth run. Leather on steel.

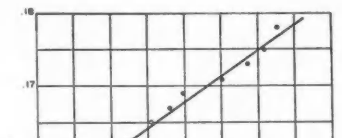

Fig. 7. Variation of coefficient of friction with logarithm of speed. Steel on steel.

Table I displays the numerical values applying to the normal force of 9582 grams. Figure 4 shows the plots of the coefficient *versus* the actual speed for the three different normal forces, and Fig. 5 shows the plots of the coefficient *versus* the logarithm of the speed. Each plotted point is the mean of 8 observations. These two sets of graphs reflect not only the characteristic direct variation of the coefficient with the logarithm of the speed, but also, consistent with Fig. 3, display the decrease of the coefficient with the increase of normal force at all speeds observed.

Figure 6 depicts the variation of the coefficient with the logarithm of the speed for leather on steel, comparing the first set of runs with the sixth set of runs made in a series of successive observations. A flat piece of sole leather weighted down with two kilograms was made to slide over the steel blade of a large two-man saw. This chart not only illustrates the fact that the logarithmic law holds for the friction between a fibrous surface and a metal surface, but also that where at least one of the surfaces is fibrous, the coefficient decreases with repeated performance. In the case of two fibrous surfaces, as wood on wood, the same was found to be true. Observations of repeated performance indicate the approach to a limiting friction curve, and also a day-to-day variation in the position of the curve, apparently reflecting variable surface conditions. These changing surface conditions are believed to be due in part to humidity, but definite study of this has not been made as yet. Each plotted point in Fig. 6 is the mean of two observations. The data for the intermediate four sets of runs are not included here.

The problem of the variation of dry sliding friction with exclusively metal surfaces presents experimental difficulties. The study of the friction of steel on steel proved to be rather baffling. Notwithstanding great care exercised in the preparation of the surfaces the results obtainable varied from time to time.

Figure 7, based on sets of runs taken with the same sliding steel surfaces on two different days with an interval of eight days, shows two plots differing somewhat from each other. Each of the plotted points is the mean of ten observations. The logarithmic law seems to apply fairly well to each case, although the increase in the value of the coefficient after a lapse of time is very marked. It appears that the friction of steel on steel is complicated by many variables, but that it is definitely not inconsistent with the law of logarithmic variation.

Figure 8 pictures the variation of the coefficient with the logarithm of the speed for lead on steel. Each of the 35 points represents the mean of 15 separate observations. Despite some drifts from the straight line it is quite apparent that the same law of variation holds for these two metallic surfaces as holds for fibrous surfaces.

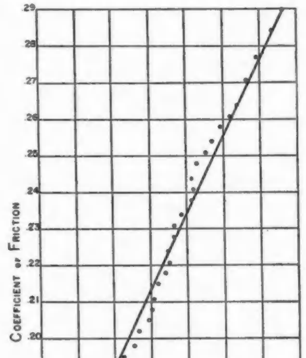

Fig. 8. Variation of coefficient of friction with logarithm of speed. Lead on steel.

The drifts of points below and above the straight line are explainable probably in terms of the irregular distributions of the gradually accumulated abraded material. There was quite a distinct wear of the softer lead surface after a large number of runs. The steel surface was wiped with a dry cotton cloth before each trip, but the lead surface was not so wiped. In the investigations that follow it will be seen that the continuous removal of the abraded material produced by the softer sliding metallic surface results in reduced friction between the surfaces.

Figure 9 shows in an interesting way the reduction in friction produced by the removal of abraded material. The upper straight solid line in the graph is based upon 15 points, each of which represents the mean of 15 observations. For each of the separate 225 observations, both surfaces were wiped with clean dry cotton cloths. Although the cloths gave no visible evidence of soiling after wiping, the results of wiping seemed to give more uniform results.

It was found that when dry-cleaning fluid was used in cleaning the surfaces a distinct reduction in friction was produced. The volatile dry-cleaning fluid perchloroethylene was used in wiping both lead and steel surfaces. For the first three lower points of the dotted line, the fluid was applied before each run and the surface vigorously wiped dry. The wiping cloths were found to be soiled slightly with the removal of the minute particles of abraded material after each run. In this connection it should be noted that when two metallic surfaces were employed, the time of the first half of the run was in many cases less than the time of the second half, thus indicating a slight deceleration. This is explainable by the gradual production of the abraded material along the path of motion by the softer of the two metals in contact, causing a gradual increase in friction along the run as the abraded material literally piled up. The next two points of the dotted line graph show the reduced speed occasioned by the nonremoval of abraded material by means of the fluid, although both surfaces were wiped. The upper seven points of the graph follow a straight line very closely. For these points no dry-cleaning fluid was used.

The solid line just above the dotted line displays a distribution of twelve plotted points; all of them fall very close to a straight line. For these twelve points no dry-cleaning fluid was used, but both surfaces were wiped after each run with dry cotton cloths. Apparently the thorough cleansing by the dry-cleaning fluid at the start of the previous set of runs was still effective in part, as this straight line is displaced considerably lower on the graph than the first one described in the figure. It should be noted that each point in the two lower plots of Fig. 9 represents the mean of 5 observations.

Results with dry-cleaning fluid corresponding exactly to those for lead on steel were found for both brass on steel and for aluminum on steel.

Figure 10 contains five plotted lines based

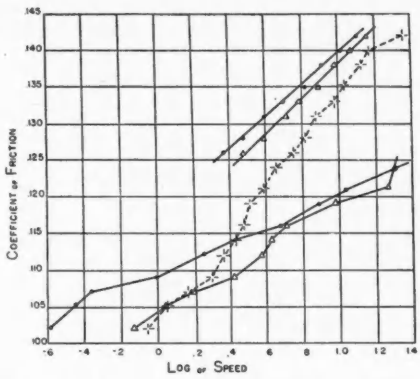

FIG. 10. Variation of coefficient of friction with logarithm of speed and with degree of cleansing of surfaces. Brass on steel.

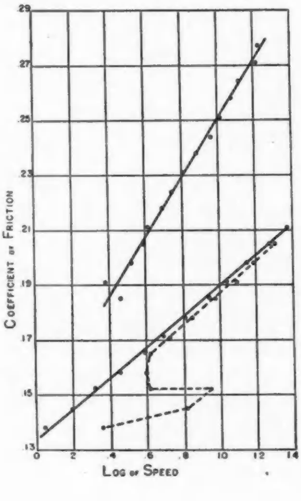

FIG. 9. Variation of coefficient of friction with logarithm of speed and with degree of cleansing of surfaces. Lead on steel.

TABLE II. Variation of coefficient of friction with speed. Brass on steel. Various preparatory treatments of surfaces; both surfaces wiped dry.

Gross friction force (grams)	Speed (centimeters per second)	Adjusted coefficient of friction	Logarithm of speed
270	2.33	0.126	0.366
275	3.00	0.128	0.478
280	3.97	0.131	0.599
285	4.99	0.133	0.698
290	6.59	0.135	0.819
295	8.07	0.137	0.907
300	10.42	0.140	1.018
305	12.73	0.142	1.105

Normal force = 2090 grams
Length of path = 100 centimeters.
8 points; each point is the mean of 25 observations.

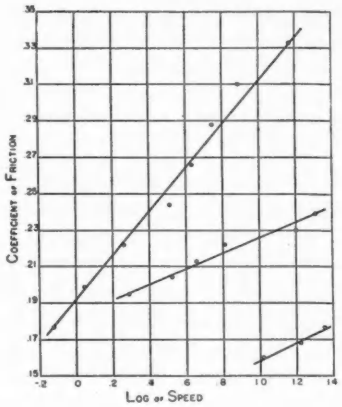

FIG. 11. Variation of coefficient of friction with logarithm of speed and with degree of cleansing of surfaces. Aluminum on steel.

upon the measurements for brass on steel. The top solid line has eight points, each of which is the mean of 25 observations. Table II displays the data for this plot. The solid line next to the top and close to it also has eight points, each of which is the mean of 20 observations. Data for these two lines were taken one week apart. Both surfaces were simply wiped with dry cotton cloths between separate runs. The dotted line with each plotted point marked by a cross was produced in the same way as the corresponding line in Fig. 9, except that for the first few points a patented "Nacto" dry-cleaning fluid was used in place of the perchloroethylene used with lead on steel. For the succeeding points in this dotted line the surfaces were wiped with dry cloths only. Each of the 18 points is the mean of 15 observations. The two lowest broken lines show the result of the application of cleaning fluid between each run. Each of the seven points in the upper line of these two is the mean of 10 observations. Each of the ten points in the lowest line is the mean of 20 observations. The data for the lowest line were taken prior to the one immediately above.

Figure 11 contains three plotted lines based upon the measurements for aluminum on steel. The top line contains eight plotted points, each of which is the mean of 10 observations. For this line neither surface was wiped after the observations had been begun. The middle plotted line contains six points, each of which is the mean of 10 observations. Both surfaces were wiped with dry cloths between each run. The effect of wiping the surfaces only with dry cloths is seen to result in decreased friction. The lowest line containing three points, each of which is the mean of 20 observations, shows the result of cleaning both surfaces thoroughly with dry-cleaning fluid before each run. The dry-cleaning fluid used in this case was perchloroethylene. A considerable reduction in friction is to be noted.

As regards the coefficient of friction between surfaces it can be seen from the above discussion that *at all times the value of the coefficient should be qualified with reference to the relative speed of the surfaces in contact.*

In the case of the brass on steel the abraded material wiped off on the cloth had a greenish cast. For aluminum on steel the cloths were soiled with gray. For the former combination the abraded material was largely wiped from the brass. For the latter, the abraded material was wiped from both surfaces with probably a little more of it coming from the steel surface than from the aluminum. However, in both cases it was probably the softer metal which was removed.

The experimental work described above, of course, represents but a beginning to the study of the problem of sliding friction under the conditions specified.

The diagrams in this article were prepared by my son, Charles T. Maney, Assistant Professor of Electrical Engineering, University of Kentucky.

All of the measurements for Fig. 9 were made by three undergraduate students, Phil Oldham, Robert Retterbush, and Byrl Short, working under the author's direction.

A Study of a New Arresting Device for Fletcher's Acceleration Apparatus

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(Received November 29, 1951)

A new mechanism for rapidly absorbing the kinetic energy of the cart in Fletcher's apparatus is described, and experiments to determine the physical basis of its operation are discussed. It is shown that the energy is absorbed by viscous flow.

RECENTLY, we have had occasion to redesign the arresting device which absorbs the energy of the accelerated cart in Fletcher's apparatus. This equipment shows the validity of Newton's second law of motion by the acceleration of a cart acted upon by the gravitational force on a suspended weight. Figure 1 shows a view of the apparatus including the new arrester.

As may be seen in the figure, the arrester consists of a $\frac{1}{4}$-in. steel rod about $3\frac{1}{2}$ in. long borne by the cart and a closed steel cylinder fastened to the track frame. At the end of the cart's run, the rod engages with the cylinder which is made about 0.002 in. larger in diameter than the rod. The arrester was originally expected to dissipate the cart's kinetic energy in heat by the compression of the gas trapped in the cylinder. The first test was a failure. The cart slammed hard against the arrester and bounced back. Application of cup grease to the plunger to improve the air seal made it no better, but when a small quantity of grease was pushed to the bottom of the cylinder so that the plunger could no longer touch the bottom without displacing the grease, the cart was brought smoothly and silently to rest with very little bouncing.

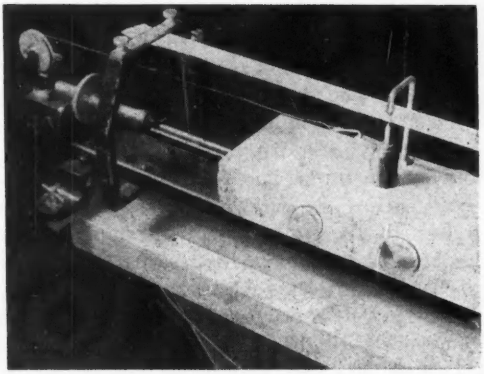

FIG. 1. Cart of Fletcher's apparatus with arrester shown at left consisting of rod attached to cart which engages with cylinder attached by simple floating mounting to track frame. Size is shown by attached 6-in. scale. Note timing marks on tape.

In order to determine more carefully the nature of the arresting mechanism, the cart was allowed to run along the track, accelerated by the force of gravity on a 100.5-g weight while a record was made by a spark timer on the waxed tape visible in Fig. 1. This recorded the cart's position every 1/30 of a second. The mass of the cart and its load was 2043.4 g. As the arrester engaged, the tape was moved sideways with the timer still running so that the record included the motion of the cart while engaged in the arrester. The data from the tape were used to plot the curve shown in Fig. 2.

The first part of the curve is very precisely parabolic, as it should be. There is no particular peculiarity discernible at the point of engagement of the arrester, 98 cm, but the point of maximum travel, 104 cm, agrees exactly with the final rest position found by applying a static load to the cart. This indicates clearly that the

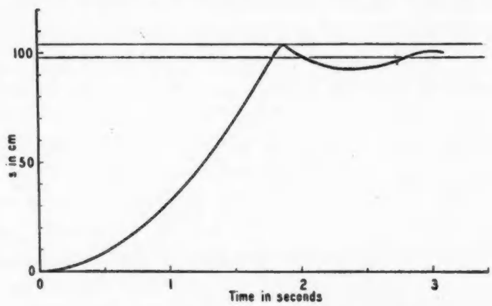

Fig. 2. Plot of data from tape shown in Fig. 1 giving position of cart as a function of time. Line at $s=104$ cm indicates position at which rod reaches bottom of air space in cylinder. Line at $s=98$ cm indicates point at which rod first engages with cylinder.

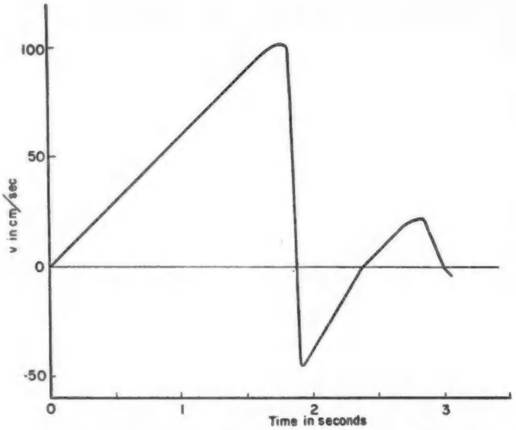

FIG. 3. Plot of cart velocity as function of time. Obtained by graphical differentiation of curve shown in Fig. 2.

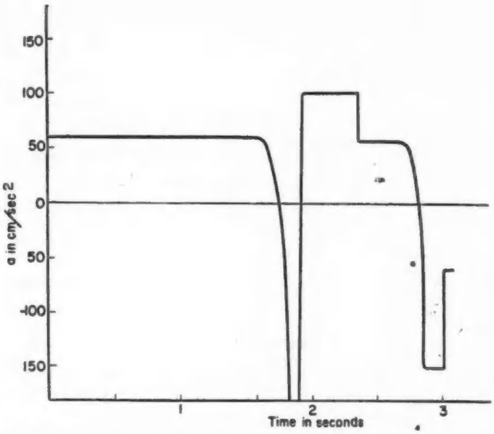

FIG. 4. Plot of acceleration of cart as function of time. Obtained by graphical differentiation of velocity function. Extreme negative value at $t=1.8$ sec is -1560 cm/sec^2.

air either escapes or is compressed to a negligible volume. The cart then reverses its motion and bounces free of the arrester, falls in again and bounces once more before the record ends, the plunger this time not going to the bottom.

The slopes of the displacement function were determined graphically, point by point, to determine the velocity as a function of time as plotted in Fig. 3. It is first linear, changing rapidly when the arrester engages and very rapidly as it hits bottom, until the maximum negative value is reached as the cart starts back out of the arrester. The velocity then increases linearly to zero, at which point there is a clear change of slope and the cycle is repeated less violently. The values of maximum and minimum velocity provide measures of the energy-absorbing ability of the arrester. On the first rebound, the energy has been reduced to 20 percent of its value before the impact, and by the time it has rebounded the second time its energy is only 1.5 percent of the initial value. We conclude that the arrester is doing its job very well indeed.

The changes of slope at zero velocity correspond to the reversals of the frictional force as may be seen even more clearly from the acceleration which was plotted by graphical determination of the slopes of Fig. 3 and is shown in Fig. 4. At $t=1.8$ sec, at the instant of hitting the bottom, the acceleration reaches the extreme value of -1560 cm/sec^2. Between 2.0 and 2.6 sec, when the cart is free of the arrester, the acceleration is greater at first, since the gravitational and frictional forces are in the same direction, and then drops when they oppose.

We conclude that the main action of the arrester comes from the dissipation of energy in the viscous flow of the grease in the annular space between the plunger and cylinder and that this action is very effective. A few details of the mechanical design may be of interest. The plunger is turned from solid $\frac{3}{8}$-in. rod, leaving a shoulder and a threaded stud to bolt to the end plate of the cart. The cylinder is turned and bored from larger stock. Its open end is flared, as may be seen in Fig. 1, so that it will pick up the plunger gently and not tend to throw the cart off the track. The flange is silver-soldered to the cylinder to keep it from falling too far out of line. The cylinder passes through an oversized hole in the frame and is retained only by a loosely fitting set screw, so that it may align itself with the plunger without applying any significant lateral force to the cart. No trouble has been experienced with carts being knocked off the track by the arrester. This was a common occurrence with the device originally provided. This new arrester should be useful in any apparatus, whether horizontal or vertical, where it is necessary to absorb the kinetic energy rapidly with as little disturbance of the apparatus as possible.

An Undergraduate Course in Radiation Physics

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(Received December 10, 1951)

A course is described dealing with the basic phenomena of radiation. Knowledge of differential equations is prerequisite. Since no appropriate textbook is available, more than thirty textbooks are mentioned from which a student may draw his material. Laboratory experiments are an important part of the course. Sources of material for these experiments are quoted.

IN the October, 1945 issue of the *American Journal of Physics*, Caswell and Gordy describe a proposed reorganization of undergraduate physics.[1] One subdivision of this reorganization suggests a course in radiation physics. It is the purpose of this paper to outline a course in radiation physics for senior undergraduate physics majors. In many liberal arts colleges broad educational requirements make it impossible for students to study separate courses in each of the topics covered. It is for such institutions that this course is primarily intended.

OBJECTIVES

The primary objectives are:

(1) A unified presentation, so far as possible, of some of the important radiation phenomena.

(2) Development of the mathematical technics common to much of physics.

(3) Encouragement of independent study.

(4) Development of laboratory skills.

The first objective requires that the instructor have a broad view of the entire course with particular awareness of those categories that are characterized by a similar mathematical formalism or that can be derived by application of a common principle.

The second objective requires the solution of differential equations and the evaluation of the arbitrary constants in terms of the initial and boundary conditions of the problem; therefore, one year of calculus and one semester of differential equations are prerequisites. A good course in electricity and magnetism is also presupposed.

[1] A. E. Caswell and Walter Gordy, Am. J. Phys. **13**, 315 (1945).

ORGANIZATION

Each of the broad topics numbered I to VIII under the heading "Subject Matter" is covered in three weekly classroom meetings of fifty minutes. In addition to the regular class meetings four evening meetings in the nature of seminars are held. These seminars, covering wave motion and sound, vector analysis, physical optics, and generalized coordinates and the method of Lagrange are designed to supplement the regular classroom meetings.

A laboratory runs concurrently with the course and cannot be divorced from it without serious loss. The laboratory experiments correlate with the classroom work and are designed to extend the knowledge of the student and to emphasize details of important aspects of the theory which can only be suggested in class. Some valuable experiments which cannot be included in the regular laboratory are done as demonstrations before the class.

No book or laboratory manual exists in which all of the topics are treated in the desired manner. As a consequence a considerable task is imposed upon the instructor in the preparation of suitable lecture notes, collateral reading assignments, problems, experiments, and demonstrations. This is not, however, without great advantage, for the content of the course can be revised readily and the level of difficulty modified easily to suit the needs of the individual class.

SUBJECT MATTER

The principle items studied are listed below:

I. *Oscillatory motion:* definitions (periodicity, frequency, etc.); free oscillation; damped oscillation; forced oscillation; coupled oscillation; resonance, energy of oscillating systems; application to pendulum, springs, galvanometers,

and electrical circuits; superposition of simple harmonic vibrations.

II. *Qualitative study of wave motion:* definition of wave motion; traveling waves; standing waves; transverse and longitudinal waves; phase velocity; group velocity; interference; modulation; dispersion; polarization; Doppler effect.

III. *Optics:* nature of light; experiments to determine the speed of light; index of refraction; reflection and refraction at plane surfaces; prisms; reflection and refraction at spherical surfaces; thin lens; compound lenses; thick lens; mirrors; aberrations; optical instruments; Huygens' principle; polarization; diffraction of light; interference of light.

IV. *Mathematical interlude:* orthogonality properties of the sine and cosine function; expansion of simple functions in Fourier series; harmonic analysis; study in rectangular coordinates of gradient of a scalar; divergence and curl of a vector; certain common vector relations involving the operator "del" (nabla).

V. *Electromagnetic waves:* Maxwell's equations in rectangular coordinates; derivation of the wave equation; solutions of the wave equation; propagation and reflection of plane waves; Poynting's vector; power flow in a plane wave; rectangular waveguides; rectangular resonator.

VI. *Sound and ultrasound:* acoustic plane waves; speed of sound in fluids; reflection and transmission of plane acoustic waves; transverse vibrations of a string; longitudinal vibrations of a rod; vibrations of a rectangular membrane; acoustic resonators; the ear; longitudinal piezoelectric vibrator; ultrasonic phenomena in gases, liquids, and solids.

VII. *Interaction of matter and radiation and survey of experiments indicating the need for quantum mechanics:* experiments to determine e/m; experiments to determine e; experiments on the scattering of alpha-particles; blackbody radiation; the photoelectric effect; the Compton effect; discrete energy levels (spectra); Stern-Gerlach experiment; electron diffraction.

VIII. *Introduction to quantum mechanics:* postulates of quantum mechanics; characteristic states and characteristic values of momentum, moment of momentum, and energy of a single particle; quantum-mechanical averages; principle of indeterminacy; free particle in a box; potential barriers.

SEMINARS

Four evening seminars introduce or supplement some of the foregoing topics. A study outline is prepared for each seminar listing the topics to be discussed with references from which the pertinent material can be obtained. For the fourth seminar references are supplemented by a set of notes giving all the necessary material together with numerous examples illustrating the theory. The content of the four seminars follows:

I. *Wave motion and sound:* speed of waves in elastic media; the equation of the wave; interference phenomena; stationary waves; refraction; reflection; diffraction.

II. *Vector analysis:* definitions; vector addition and subtraction; coordinate systems; vector multiplication; vector differential operator; the gradient of a scalar; divergence of a vector; curl of a vector.

III. *Physical optics:* interference and the principle of superposition; phase difference in terms of path lengths; reflection of the E vector from a surface; reflection from thin films; Newton's rings; nonreflecting film; distinction between Fraunhofer and Fresnel diffraction; diffraction from a single slit; Fresnel zones; Raleigh criterion; resolution of a grating and of a prism.

IV. *Generalized coordinates and the method of Lagrange:* degrees of freedom and generalized coordinates; total energy in Cartesian rectangular coordinates; transformation of total energy to generalized coordinates; the differential equations of motion by the method of Lagrange; selected problems for solution—free particle, harmonic oscillator, simple pendulum, conical pendulum, ideal Atwood's machine.

LABORATORY

The laboratory consists of twenty-four experiments designed to be completed in thirty three-hour laboratory periods. Most of the apparatus is available generally even in small physics laboratories. Special apparatus required can be constructed easily with shop facilities of the simplest kind. Apparatus for some experi-

ments was adapted from government surplus equipment.

The experiments are listed:

1. The simple pendulum: adapted from two experiments described in this journal. Refer to (1) and (2) in the list of useful articles.

2. Coupled pendulums: refer to (3) in the list of useful articles.

3. The cathode-ray oscillograph: use of the oscillograph as a laboratory instrument; phase and frequency comparison; calibration as a voltmeter.

4. The index of refraction of a prism: use of the Gauss' eyepiece; adjustment of the spectrometer; determination of the index of refraction of a prism for selected wavelengths.

5. Index of refraction of transparent substances: determination of the index of refraction with a traveling microscope; determination of the index of refraction of liquids by the method of total reflection using the spectrometer and two like prisms.

6. Constant-deviation spectrometer: measurement of the wavelengths of lines in helium, hydrogen, and mercury spectra; spectral series by the Bohr theory; Cauchy dispersion formula.

7. Thin lenses: focal lengths of converging and diverging lenses; index of refraction of the lens from spherometer measurements; object-image distances; magnification. (Both Gaussian and Newtonian formulas are used in this experiment.)

8. Lens systems and the thick lens: the nodal slide; location of cardinal points; image-object relation; transverse magnification. (The Newtonian equation is used exclusively.)

9. The plane diffraction grating: measurement of wavelength; resolving power of grating; dispersion of grating.

10. Lens testing on a student spectrometer: Refer to (17) in the list of useful articles.

11. The Michelson interferometer: adjustment of the interferometer; measuring the thickness of a thin sheet of transparent material.

12. Measurement of wavelength with a Fresnel biprism and with Fresnel's mirrors: the eyepiece micrometer; measurement of the angular separation between the virtual sources produced by the biprism; adjustment of the Fresnel mirrors; measurement of wavelengths.

13. Production of electromagnetic oscillation: vacuum-tube oscillators utilizing conventional tuned LC, transmission line, and cavity-type resonance circuits; effect of a change in the circuit tuning on the frequency of oscillation.

14. Oscillation characteristics of a reflex klystron (723A/B): output of the oscillator for various shell voltages as the repeller voltage is varied.

15. Wavelength and frequency measurements in the microwave region of 3 and 10 cm: wavelength in the guide; wavelength in air; radio fading simulated at 10 cm.

16. Electric field in a circular aperture for wavelengths of 3 and 10 cm: variation of the E field for both planes of polarization.

17. Diffraction pattern for various apertures at wavelengths of 3 and 10 cm: exploration of the field behind single slots, multiple slots; square and circular apertures; Babinet's principle.

18. Interference of radiation from two-point sources of wavelength 10 cm.

19. Lecher wires: inductance and capacity of lines; standing waves on a Lecher wire system; open lines; short circuit lines; frequency of oscillation.

20. Antenna arrays: parasitic antennas; patterns with a single parasitic element; patterns with several elements.

21. Velocity of sound: Kundt's apparatus; vibrating string (sonometer); Melde's experiment.

22. Acoustical resonators: resonators with multiple openings; resonance with a bottle.

23. The vacuum photoelectric tube: characteristics of the tube; color response.

24. Laue x-ray diffraction: refer to (22), (23), (24), and (25) in the list of useful articles.

DEMONSTRATIONS

The following experiments are done as classroom demonstrations and are designed to illustrate certain aspects of instruction rather than to produce sensational effects.

1. Dependence of the period of a pendulum upon the amplitude of the oscillation.

2. Response of series RLC and transformer-coupled circuits to "square waves" of voltage.

3. Lissajous figures produced electrically and mechanically.
4. Beats and modulation.
5. Aberrations of lenses.
6. Interference patterns of various apertures and obstacles.
7. Polarization of electromagnetic waves.
8. Passage of electromagnetic waves through dielectric sheets.
9. Focusing of electromagnetic waves.
10. Radiation pattern of horns.

USEFUL ARTICLES

The *American Journal of Physics* is a rich source of material for a course such as outlined. Following is a list of articles used as a basis for experiments, demonstrations, and classroom discussions which is of continuing usefulness. Articles are listed approximately in the order in which they are used.

1. Philip A. Constantinides, "An experimental study of simple harmonic motion." Am. J. Phys. **7**, 417 (1939).
2. Albert Burris and W. J. Hargrave, "A simple pendulum energy experiment," Am. J. Phys. **12**, 215 (1944).
3. Leonard O. Olsen, "Coupled pendulums: an advanced laboratory experiment," Am. J. Phys. **13**, 321 (1945).
4. Gwilym E. Owen, "The frequency produced by the combination of two vibrations of nearly equal frequency," Am. J. Phys. **7**, 177 (1939).
5. Robert W. Leonard, "An interesting demonstration of the combination of two linear harmonic vibrations to produce a single elliptical vibration," Am. J. Phys. **5**, 175 (1937).
6. H. N. Walker and P. Greensteen, "Direct current transients with the square wave generator," Am. J. Phys. **10**, 198 (1942).
7. L. B. Ham, "Loudness and intensity," Am. J. Phys. **9**, 213 (1941).
8. Herbert Jehle, "Phase and group velocity," Am. J. Phys. **14**, 47 (1946).
9. S. Millman and M. W. Zemansky, "Wave velocities in elementary physics," Am. J. Phys. **13**, 250 (1945).
10. Francis E. Fox, "Demonstration of the Doppler effect," Am. J. Phys. **12**, 228 (1944).
11. G. F. H. Harker, "Doppler effect when both source and observer are in motion," Am. J. Phys. **12**, 175 (1944).
12. J. O. Perrine, "The Doppler echo and Doppler effect," Am. J. Phys. **12**, 23 (1944).
13. Harold K. Schilling, "Acoustic experiments in the teaching of optics," Am. J. Phys. **6**, 156 (1938).
14. Wesley M. Roberds, "Some simple experiments on optical resolution," Am. J. Phys. **5**, 182 (1937).
15. F. A. Molby, "Diffraction of light, an experimental demonstration," Am. J. Phys. **5**, 78 (1937).
16. Lawrence E. Kinsler, "Imaging of underwater objects," Am. J. Phys. **13**, 255 (1945).
17. Everett F. Cox, "Lens testing on a student spectrometer," Am. J. Phys. **6**, 153 (1938).
18. Arthur S. Jensen, "Lens aberrations—a classroom demonstration," Am. J. Phys. **13**, 113 (1945).
19. C. L. Andrews, "Graph of the lens equation in three variables," Am. J. Phys. **11**, 292 (1943).
20. J. H. McMillen, "A course in applied spectroscopy," Am. J. Phys. **11**, 126 (1943).
21. Gordon Ferrie Hull, Jr., "Experiments with UHF waveguides," Am. J. Phys. **13**, 384 (1945).
22. A. H. Weber, J. F. McGee and K. F. Gerhard, "An undergraduate experiment in Laue x-ray diffraction," Am. J. Phys. **5**, 279 (1937).
23. A. P. R. Wadlund, "A portable Laue spot camera," Am. J. Phys. **6**, 103 (1938).
24. William R. McMillan, "Equipment for elementary Laue x-ray studies," Am. J. Phys. **13**, 327 (1945).
25. Willis C. Campbell, "A simple x-ray diffraction camera," Am. J. Phys. **15**, 409 (1947).
26. Arthur H. Compton, "The scattering of x-ray photons," Am. J. Phys. **14**, 80 (1946).
27. Austin J. O'Leary, "Two elementary experiments to demonstrate the photoelectric law and measure the Planck constant," Am. J. Phys. **14**, 245 (1946).
28. W. V. Houston, "The physical content of quantum mechanics," Am. J. Phys. **5**, 49 (1937).

BOOKS

Because no textbook is used, it is necessary that a sufficient number of books be available to provide both collateral reading assignments and supplementary reference material. A large number of excellent books is currently available. The list that follows, adequate for the above purpose, may be readily modified or extended.

1. Arguimbau, *Vacuum Tube Circuits* (John Wiley & Sons, Inc., New York, 1948).
2. Bergmann, *Ultrasonics* (John Wiley & Sons, Inc., New York, 1948).
3. Bohm, *Quantum Theory* (Prentice-Hall, Inc., New York, 1951).
4. Born, *Atomic Physics* (G. E. Stechert & Company, New York, 1936).
5. Bronwell and Beam, *Theory and Application of Microwaves* (McGraw-Hill Book Company, Inc., New York, 1947).
6. Carlin, *Ultrasonics* (McGraw-Hill Book Company, Inc., New York, 1949).
7. Churchill, *Fourier Series and Boundary Value Problems* (McGraw-Hill Book Company, Inc., New York, 1941).
8. Dushman, *The Elements of Quantum Mechanics* (John Wiley & Sons, Inc., New York, 1938).
9. Emery, *Ultra-High Frequency Radio Engineering* (Macmillan Company, New York, 1944).
10. Hildebrand, *Advanced Calculus for Engineers* (Prentice-Hall Inc., New York, 1949).

11. Jauncey, *Modern Physics* (D. Van Nostrand and Company, Inc., New York, 1937).

12. Jenkins and White, *Fundamentals of Optics* (McGraw-Hill Book Company, Inc., New York, 1950)

13. Jordon, *Electromagnetic Waves and Radiating Systems* (Prentice-Hall, Inc., New York, 1950).

14. Kinsler and Fry, *Fundamentals of Acoustics* (John Wiley & Sons, Inc., New York, 1950).

15. Lass, *Vector and Tensor Analysis* (McGraw-Hill Book Company, Inc., New York, 1950).

16. Leprince Ringuet, *Cosmic Rays* (Prentice-Hall, Inc., New York, 1950).

17. Lindsay, *Concepts and Methods of Theoretical Physics* (D. Van Nostrand and Company, Inc., 1951).

18. Lindsay, *Physical Mechanics* (D. Van Nostrand and Company, Inc., New York, 1933).

19. Marchand, *Ultra-High Frequency Transmission and Radiation* (John Wiley & Sons, Inc., New York, 1947).

20. MIT Radar School Staff, *Principles of Radar* (McGraw-Hill Book Company, Inc., New York, 1946).

21. Monk, *Light* (McGraw-Hill Book Company, Inc., New York, 1937).

22. Page, *Introduction to Theoretical Physics* (D. Van Nostrand and Company, Inc., New York, 1935).

23. Ramo, *Introduction to Microwaves* (McGraw-Hill Book Company, Inc., New York, 1945).

24. Richardson, *Sound* (Longmans, Green and Company, New York, 1947).

25. Robertson, *Introduction to Physical Optics* (D. Van Nostrand and Company, Inc., New York, 1941).

26. Rojansky, *Introductory Quantum Mechanics* (Prentice-Hall, Inc., New York, 1942).

27. Sarbacher and Edson, *Hyper and Ultra-High Frequency Engineering* (John Wiley & Sons, Inc., New York, 1943).

28. Sears, *Principles of Physics, III* (*Optics*) (Addison-Wesley Press, Inc., Cambridge, Massachusetts, 1948).

29. Skilling, *Fundamentals of Electric Waves* (John Wiley & Sons, Inc., New York, 1948).

30. Sokolnikoff, *Advanced Calculus* (McGraw-Hill Book Company, Inc., New York, 1939).

31. Sokolnikoff and Sokolnikoff, *Higher Mathematics for Physicists and Engineers* (McGraw-Hill Book Company, Inc., New York, 1940).

32. Stewart and Lindsay, *Acoustics* (D. Van Nostrand and Company, Inc., New York, 1930).

33. Stranathan, *The "Particles" of Modern Physics* (The Blackiston Company, Philadelphia, Pennsylvania, 1942).

ACKNOWLEDGMENT

The author desires to express his gratitude to Dr. Kenichi Watanabe now of the Air Force Research Laboratories in Cambridge, Massachusetts, for helpful suggestions made regarding the course described.

The [ancient] atomists and their ideas did not emerge suddenly out of nothing, they were preceded by the great development that began with Thales of Miletus (floruit 585 B.C.) more than a century earlier; they continue the awe-inspiring line of Ionian physiologoi. Their immediate predecessor in this line was Anaximenes, whose principal doctrine consisted in underlining the all-importance of "rarefaction and condensation." From a careful consideration of everyday experience he abstracted the thesis that every piece of matter can take on the solid, the liquid, the gaseous and the "fiery" state; that the changes between these states do not imply a change of nature, but are brought about geometrically, as it were, by the spreading of the same amount of matter over a larger and larger volume (rarefaction), or—in the opposite transitions—by its being reduced or compressed into a smaller and smaller volume. This idea is so absolutely to the point that a modern introduction into physical science could take it over without any relevant change. Moreover it is certainly not an unfounded guess, but the outcome of careful observation.

If you try to assimilate Anaximenes' idea, you naturally come to think that the change of properties of matter, say on rarefaction, must be caused by its parts receding at greater distances from each other. But it is extremely difficult to accomplish this in your imagination, if you think of matter as forming a gapless continuum.—E. SCHRÖDINGER, Science and Humanism (1951).

Conductivity Crystal Counters*

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(Received December 20, 1951)

This is a brief review article on conductivity crystal counters in which their main properties and basic principles of operation are described. Their present position in the field of nuclear instrumentation is discussed, it being concluded that there are still many serious problems to be solved before the counters can be of practical use. Some of these problems and the present methods of tackling them are surveyed.

THE recent interest in conductivity crystal counters[1] started in 1945 when van Heerden published a thesis[2] describing the successful detection of single α-particles and γ-photons by single crystals of silver chloride. This new method of detecting nuclear particles immediately received much attention and there quickly followed numerous publications giving, mostly, the qualitative results of searches made in the hope of finding the most suitable crystals for use as counters. It soon became apparent, however, that the counting efficiencies and sensitivities of crystals varied considerably from specimen to specimen of the same substance. Also, it was found that the charge pulses caused by prolonged exposure to ionizing radiations grew smaller as the bombardment proceeded. Probably these effects are mainly responsible for the fact that conductivity crystal counters have not yet become the useful research tool that they initially promised to be.

MECHANISM OF THE CRYSTAL COUNTER

In order to understand some of the factors governing the behavior of a crystal counter it is necessary to describe its mechanism. It is most convenient to use the the energy-band diagram as shown in Fig. 1 where *A* represents the outermost completely full electronic energy band associated with the insulating crystal, and *B* is the next higher band which is normally completely empty. As there are normally no vacant levels in *A*, electronic conduction cannot take place. If, however, an electron is removed from *A* thereby leaving behind a "positive hole" at *C*, it might be possible for an electron in a neighboring level in the band *A* to occupy the vacant site, thus leaving a new hole in its previous level. By continued repetitions of this process, the positive hole can be regarded as migrating through the crystal lattice. In some crystals, for example, diamond, this effect is known to take place, through in others, such as AgBr, the holes are stationary.

The electron that was ejected from *C* may reach the conduction band *B* in which case it is completely free to wander through the crystal lattice. The effect of placing an electric field across the crystal will be to make the free electron move in the general direction of the anode and similarly, the positive hole will move toward the cathode.

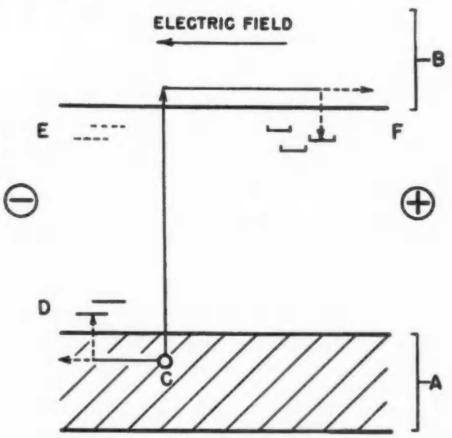

FIG. 1. The energy-band scheme for an insulating crystal showing diagrammatically the basic processes that take place in a crystal counter.

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[1] Detailed review articles on this subject have been written by R. Hofstadter: Nucleonics **4**, No. 4, 2 (1949); **4**, No. 5, 29 (1949); and Proc. Inst. Radio Engrs. **38**, 726 (1950).
[2] P. J. van Heerden, Utrecht dissertation, 1945. See Physica **16**, 505 and 517 (1950).

Basically, the action of the counter can now be given. An ionizing radiation will create electron-hole pairs, which under the influence of an electric field will separate, the electrons moving towards the anode; if the holes move, they will drift towards the cathode. This movement of charges in the crystal will cause changes in the charges induced on the electrodes which, after suitable amplification, can be recorded as a charge pulse.

In the ideal crystal the electrons and holes will move until they meet the metal electrodes, the electrons entering the anode and the holes recombining with an electron from the cathode (assuming favorable potential conditions at the junctions). If the holes do not move a positive space-charge field will form in the crystal as the bombardment proceeds. This simple picture must now be modified as in practice no crystal is ideal. Chemical impurities may be present and there can also be physical imperfections in the periodicity of the lattice. Examples are: (i) foreign ions or an excess of one of the components of the lattice; these can occupy either ordinary lattice sites or interstitial positions, (ii) vacant lattice sites, which can also be occupied by electrons (F- and F'-centers), (iii) cracks, dislocation planes, regions of mechanical strain, or boundaries between the microscopic mosaic blocks that sometimes make up a crystal as a whole. In Fig. 1, the effect of impurities may often be represented by local discrete energy levels at D with associated excitation levels at E. The effect of imperfections can be represented by local discrete levels at F which are normally vacant. With these added complications various processes are now possible. In wandering through the lattice, the positive hole may meet an impurity center D where it can become trapped by attracting an electron from the center to the band A. It is also probable that crystal imperfections can end the wanderings of a hole. Similarly, an electron in the conduction band can either (i) meet and combine with another positive hole, (ii) jump into a vacant excitation level E and hence to the ground state, possibly emitting a light quantum in doing so, or (iii) become held in a trapping state.

Concluding, the fate of freed electrons and holes can be either: (i) to reach the electrodes,

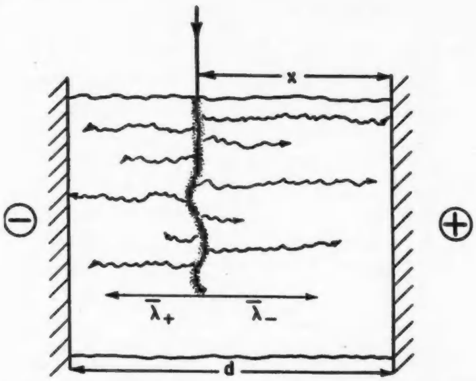

FIG. 2. Diagram of the case where the incident radiation travels parallel to the electrodes. The distances $\bar{\lambda}_-$ and $\bar{\lambda}_+$ represent the mean distances traveled by the electrons and the positive holes, respectively.

(ii) to recombine with other holes and electrons after wandering for some distance, or (iii) to reach trapping states situated throughout the crystal lattice.

FORMULATION OF THE COUNTER CHARACTERISTICS

It is informative to formulate how the observed pulse height caused by a particle of given energy depends on the applied field and the physical condition of the crystal, the crystal being considered as free from any space charge fields. Only the case shown in Fig. 2 will be dealt with, where the incoming particle travels in a direction perpendicular to the electric field and produces ionization in a region approximately parallel to the electrodes. The second case where the ionizing particle travels parallel to the field can be dealt with in an analogous manner. Let the electron-hole pairs be formed at a distance x from the anode. Owing to the traps existing in the crystal (their spatial distribution is assumed to be homogeneous), the electrons and holes will drift varying distances towards the electrodes. Let λ_- and λ_+ represent the mean-free paths of the electrons and holes respectively, with reference to trapping centers. Then

$$\lambda_- = k_- F\tau_- \quad \text{and} \quad \lambda_+ = k_+ F\tau_+, \tag{1}$$

where k_-, k_+ are the drift mobilities of the electrons and holes, F is the applied field, and τ_-, τ_+ are the mean free times spent by the electrons

FIG. 3. The theoretical variation of the pulse height q with applied field strength (λ/d is directly proportional to the field) for the case where all the ionization occurs close to an electrode.

and holes before being trapped. If all the electrons and holes terminate their paths inside the crystal, it can be shown that the charge pulse recorded is then

$$q = ne(\lambda_+ + \lambda_-)/d, \tag{2}$$

where n is the number of electron-hole pairs formed, e is the electronic charge, and d is the separation between the electrodes. If all the electrons and holes reach the electrodes, the charge pulse is $q = ne$. However, in practice, the intermediate case usually exists where only some of the electrons and holes reach their respective electrodes. In this case, it can be shown that the new mean-free paths $\bar{\lambda}_-$ and $\bar{\lambda}_+$ are now given by[3]

$$\bar{\lambda}_- = \lambda_-[1 - \exp(-x/\lambda_-)],$$
$$\bar{\lambda}_+ = \lambda_+\{1 - \exp[-(d-x)/\lambda_+]\}.$$

Hence, the recorded pulse height is given by

$$q(d/ne) = \lambda_-[1 - \exp(-x/\lambda_-)] + \lambda_+\{1 - \exp[-(d-x)/\lambda_+]\}. \tag{3}$$

The general conclusions that can be drawn from this equation are that: (i) for a constant value of x, the pulse height is proportional to the number of electron-hole pairs formed and is therefore most likely to be proportional to the energy lost in the crystal by the ionizing radiation, (ii) if x is small, the charge pulse is caused mainly by the motion of the positive holes and vice versa if $x \simeq d$. This case also occurs when radiation of very low penetrating power strikes the crystal through the electrodes and consequently, separate studies can be made of the motion of the electrons and holes simply by reversing the field. This technique has been used with success by McKay.[4] Figure 3 shows the variation of pulse height with applied field strength for the case where the ionization occurs close to an electrode (the parameter λ/d being directly proportional to the applied field). Then

$$q/ne = (\lambda_\pm/d)[1 - \exp(-d/\lambda_\pm)]. \tag{4}$$

It is seen that when λ is rather larger than the crystal dimension d, the pulse height is roughly independent of the field.

By inspection of Eqs. (3) and (4) it is clear that the optimum pulse height is obtained when the mean-free paths λ_-, λ_+ are a maximum and also, when n is as large as possible, i.e., when all the energy of the incoming particle is used up in creating free electrons and holes. As the success or failure of a crystal counter is decided mainly by the factors affecting these quantities it is advantageous to consider them in more detail.

THE REQUIREMENTS OF COUNTING CRYSTALS

The mean-free path of an electron with respect to trapping centers (λ) is directly related to the mean-free path with respect to the lattice vibrations (l) by the equation[5]

$$\lambda/F = el/6kTP\sigma, \tag{5}$$

where T is the absolute temperature, k is Boltzmann's constant, P is the density of trapping states, and σ is the cross section for electron trapping. Estimates of l can be made for various crystals using the theory of Fröhlich and Mott[6] and, assuming somewhat arbitrary values for the other quantities in the equation, estimates of λ can be made. This has been done by Hofstadter[1] and his results are listed in Table I (all at 77°K). The first three crystals in the table have been observed to count; but in spite of an intensive investigation, Hofstadter has not had any success with LiF. However, following Hofstadter, the statement that "a certain

[3] N. F. Mott and R. W. Gurney, *Electronic Processes in Ionic Crystals* (Oxford University Press, New York, 1948), p. 122.

[4] K. G. McKay, Phys. Rev. **74**, 1606 (1948); and **77**, 816 (1950).

[5] See reference 3, p. 131.

[6] H. Fröhlich, and N. F. Mott, Proc. Roy. Soc. (London) **A171**, 496 (1939).

crystalline substance will not detect ionizing radiation" must be qualified considerably. Many factors may be present that can affect the counting properties to an unknown extent; for example, "(i) the material may neither be a single crystal nor made up of large crystal blocks, (ii) impurities may be present, (iii) an amplifier may have been used which does not furnish the optimum signal-to-noise ratio, allowing pulses to be masked by the thermal noise of the amplifier, (iv) particular conditions, such as annealing, low temperature, good electrode contact, good surfaces, absence of polarization, etc., may not be attainable under the conditions of the experiment, (v) unsuspected crystal imperfections and trapping centers may exist, (vi) the 'right' particle to give the largest pulse may not have been used," since the response may vary considerably according to the character of the ionizing radiation.

From the above remarks and an inspection of Eq. (5) it is obvious that the values of P and σ will have a large bearing on the counter behavior. Comparatively little is known about the nature of trapping states in crystals, especially as regards their trapping cross section. However, the density of traps possibly varies considerably with the degree of imperfection of the crystal. Consequently, efforts to make the crystal less imperfect have been made, meeting with a fair amount of success. There are reports of freshly made crystals showing no response at all to ionizing radiations until after they have been given suitable heat treatment thereby removing to a certain extent some of the mechanical strain formerly present.[7] As regards the presence of foreign impurities as trapping centers, it seems necessary, in general, to keep them to a minimum. However, successful results have been obtained with mixed crystals of (i) LiBr and AgBr, (ii) NaCl and AgCl (up to as much as 15 percent by weight of NaCl) and (iii) TlBr and TlI.[1] It might also be noted that van Heerden obtained a much better response in AgCl crystals when exposed to α-particles after a new surface had been uncovered by turning off the old in a lathe.

We turn now to a consideration of n, the number of electron-hole pairs formed. The incident energy may be given up to the crystal by various processes: (i) in creating free electrons and holes, (ii) by radiation in the lattice field, (iii) in creating excitons, and (iv) in creating phonons when there is efficient coupling between the particle energy and the acoustic vibrational modes of the crystal lattice. Thus, when a crystal has not been found to detect radiation (even though it apparently has a suitable mean-free path for electrons, as in the case of LiF), it is possible that much of the particle energy is being used up in creating excitons or phonons. An exciton may be considered as a partially ionized electron-hole pair that is free to wander through the lattice and can therefore transfer energy but not charge. The fate of an exciton may be: (i) to recombine, or (ii) to give up its energy in creating a free electron and hole at some point in the lattice where suitable conditions occur (for example, an impurity center). Clearly, such additional processes can greatly complicate the action of the counter, and as yet there is no theoretical treatment whereby any estimate may be made of the energy that is eventually used in creating free electrons and holes. Experimentally, it has been found so far that even in the most favorable cases, as much as half of the incident energy is used up in processes other than the creation of free electrons and holes.

KNOWN COUNTING SUBSTANCES

It will be appreciated that because of the detailed requirements that must be fulfilled if a crystal is to be suitable for use as a counter, it is by no means simple to predict whether a given substance will respond. Consequently, the searches made so far for counting materials have been almost entirely empirical. Large numbers of crystals have been tried, particularly by

TABLE I. Mean free paths of electrons in various halide crystals (all at 77°K).

Substance	λ (cm)	Substance	λ (cm)
AgCl	11.0	KCl	0.28
AgBr	2.1	KBr	0.07
TlBr	6.2	NaCl	0.60
LiF	>10.0		

[7] R. Hofstadter, Phys. Rev. 72, 1120 (1947).

Ahearn,[8] though the number of successful substances is relatively few. Such statements, of course, must always be qualified in view of the remarks made earlier. For example, for some time crystals of NaCl and S had been investigated without success until recently. Van Heerden, in his original paper, reported negative results with diamond. Since then, however, this substance has been found to be one of the best for counting purposes. It will suffice then to give here only a list of those materials which have been found to show any marked response to radiations; they are: AgCl,[2,9] AgBr,[10] TlBr−TlI (mixed crystals),[7] NaCl,[11] ZnS,[12] diamond,[4,13–17] S,[18] CdS,[19–22] and solid and liquid argon.[23,24] Effects have been observed in several other substances though too small to be useful. Finally, germanium semiconducting crystals have been shown to be useful under certain conditions.[25] Some of the more interesting features of various crystals will now be described.

Silver and thallium halides were among the first successful counters. Crystals prepared synthetically have to be very pure and free from strain in order to count. Too high an impurity content can give rise to an electronic conductivity. Since at room temperatures, electrolytic conductivity can also take place, the counters can only be used at low temperatures and this is obviously a serious practical handicap. AgCl crystals have been reported to respond to α- and β-particles, γ- and x-photons. Using a mixed crystal of LiBr and AgBr, neutrons have been successfully detected using the Li(n, α) reaction to liberate an **α-particle** inside the crystal.[26]

ZnS has been shown to detect α-particles, though this work has not been followed up. On the other hand, considerable investigation has been made into the counting properties of diamond. Both of these crystals have the advantage that they can be used at room temperatures. Diamond has been shown to detect α- and β-rays and also γ-photons. Obviously, only natural specimens have been used and consequently, the purity and physical perfection varies greatly between crystals. The counting properties also vary considerably from specimen to specimen. In the past a considerable amount of investigation into the physical properties of diamond has been made. It has been customary to divide diamonds according to their properties into types *I* and *II*:[27] the type *I* diamonds strongly absorb ultraviolet radiation of frequency less than about 3000A and infrared radiation of wavelength around 8μ, and they exhibit very little or no photoconductivity. On the other hand, type *II* diamonds are transparent to the 8μ-radiation and also to the ultraviolet down to a wavelength of about 2250A. Further, they show a large photocurrent. Finally, it is of interest to add that from x-ray studies of diamond, Lonsdale has concluded that really good type *I* crystals show near perfection in their lattice structure whereas type *II* diamonds appear to be laminated or mosaic.[28] However, the boundary between the two types, if it exists at all, is very vague; in fact, most diamonds seem to be a mixture of the two types in varying proportions.

There are two schools of thought as to the physical reasons for the varying properties of diamond. Hofstadter and others support the view that the imperfections in the crystal lattice are responsible for the differences. Raman, on the other hand, has proposed that the crystal can consist of the intermingling of tetrahedral and octahedral lattices, such intermingling giving rise to the variation in properties.[29] It

[8] A. J. Ahearn, Phys. Rev. **75**, 1966 (1949).
[9] Hofstadter, Milton, and Ridgway, Phys. Rev. **72**, 997 (1947).
[10] K. A. Yamakawa, Phys. Rev. **82**, 522 (1951).
[11] H. Witt, Z. Physik **128**, 422 (1950).
[12] A. J. Ahearn, Phys. Rev. **73**, 523 (1948).
[13] Wooldridge, Ahearn, and Burton, Phys. Rev. **71**, 913 (1947).
[14] Friedman, Birks, and Gauvin, Phys. Rev. **73**, 186 (1948).
[15] A. G. Chynoweth, Phys. Rev. **76**, 310 (1949).
[16] R. K. Willardson and G. C. Danielson, Phys. Rev. **77**, 300 (1950).
[17] A. G. Chynoweth, Phys. Rev. **83**, 254 (1951); **83**, 264 (1951).
[18] M. Georgesco, Compt. rend. **228**, 383 (1949).
[19] R. Frerichs, Phys. Rev. **76**, 1869 (1949).
[20] R. Frerichs, Phys. Rev. **72**, 594 (1947).
[21] H. Kallman and R. Warminsky, Ann. Physik. **4**, 69 (1948).
[22] R. Frerichs and R. Warminsky, Naturwiss. **33**, 251 (1946).
[23] N. Davidson and A. E. Larsh, Phys. Rev. **74**, 220 (1948); **77**, 706 (1950).
[24] A. W. Hutchinson, Nature, **162**, 610 (1948).
[25] K. G. McKay, Phys. Rev. **76**, 1536 (1949).
[26] K. A. Yamakawa, Phys. Rev. **75**, 1774 (1949).
[27] Robertson, Fox, and Martin, Trans. Roy. Soc. (London) **A232**, 463 (1934).
[28] K. Lonsdale, Phys. Rev. **73**, 1467 (1948).
[29] C. V. Raman, "Symposia on diamond," Proc. Indian Acad. Sci. **19A** (1944); **24A** (1946).

will suffice to say here that no definite conclusions have yet been reached and in particular, the reason for the variation in counting efficiency is still uncertain. There are only meager reports of attempts to correlate the counting efficiency with the other physical properties though in general it appears that type *II* crystals are the more efficient. Friedman, Birks, and Gauvin[14] have claimed that the comparatively few diamonds that are ultraviolet-transparent are likely to be better for detecting γ-photons. Diamonds that respond to α-particles are more frequently found; though again, it is by no means certain why some crystals will detect α-particles and not γ-photons.

Cadmium sulfide crystals have received much attention on account of their remarkable properties but it will be necessary to refer elsewhere for detailed descriptions. Kallman divides CdS crystals into two types: nonluminescent and luminescent. The nonluminescent crystals behave in a similar fashion to the ordinary conduction counter such as AgCl, the voltage pulse being of the same order of magnitude and exhibiting both a fast rise time and a rapid decay. The luminescent crystals, on the other hand, have been reported to give a voltage pulse of several volts when struck by a single α-particle, though the rise and decay times are very much longer than in the other type. The mechanism of such a counter must clearly be more elaborate than that described earlier and Kallman explains it by postulating that electrons are free to remain in the conduction band for some considerable time. For this to happen he assumes that the positive holes left by electrons freed by heat or irradiation with light or charged particles are free to migrate and eventually become held in trapping centers. The free electrons that reach the anode may enter it since the Fermi level of the metal is usually a little lower than the lowest levels in the conduction band. As this process continues, a negative space charge will build up in the metal and a positive space charge in the adjacent parts of the crystal. This electrical double layer will reduce the potential energy of those electrons in the crystal near the electrodes and the process will reach equilibrium when the lowest level of the conduction band is at about the same level as the Fermi level of the metal. On applying a field to the crystal, electrons will be able to pass freely across the electrode-crystal boundaries in both directions; in brief, the crystal is in a conducting condition. Excitation of the crystal by ionizing radiations can trigger off this current and consequently a relatively large charge pulse can ensue. The pulse can extend over an appreciable time (a conduction current has been observed more than 0.1 sec after the original excitation using α-particles). This secondary current will eventually stop since some electrons will become trapped. CdS crystal counters have successfully detected α-, β-, and γ-rays, and individual 45-kev x-ray quanta.

The possibilities of using argon in both solid and liquid form have been investigated, response being obtained for α-, β-, and γ-rays. In pure liquid argon electron trapping appears to be small and there is the further advantage that no polarization can occur. However, pulse heights for α-particles are not large since there is apparently considerable recombination in the region of ionization. The fact that the ionization density is not so great for γ-rays probably explains the relatively larger pulses in the latter case. In solid argon, Hutchinson found that the mean energy required to form an electron-hole pair by β-particles was only 2 ev compared with 25 ev in liquid argon. It is unlikely that the energy gap is as low as this and Hutchinson assumes that multiplication is responsible for the low value.

ADVANTAGES OF CONDUCTION COUNTERS

The possible advantages that the crystal counter may possess over other types of nucleon detectors will now be considered. The important properties are: (i) stopping power, (ii) speed of response, (iii) proportionality, (iv) sensitivity, and (v) efficiency. Later, some of the disadvantages will be described.

The advantages of the conductivity counter must be considered in comparison with the merits of gas-filled detectors or its close relative, the scintillation counter. We shall not consider indirect detection methods such as the photographic emulsion or the Wilson cloud chamber. The conduction counter may be regarded as the solid counterpart of the gas-filled ionization

chamber. Obviously, volume for volume, the stopping power of the solid counter is by far the greater and consequently, it is likely to be more suitable for the detection of radiation of high penetrating power (γ-rays and high energy charged particles). For example, a 1-Mev β-particle can be stopped in one or two mm of crystal whereas its path length in air at atmospheric pressure is about 3 meters. Large pulses can be obtained even with γ-rays in crystals a few mm thick. Thus the relatively high stopping power of the crystal for high energy radiations enables it to be used in a small volume (i.e., permits good geometry in experiments). However, the same remarks apply equally well to the scintillation counter.

The speed of response of the counter is mainly governed by the time spent by the freed electrons and holes before they reach the electrodes or become firmly trapped. When freed, the charges move in a kind of Brownian motion on which is superimposed a drift towards their respective electrodes. The mobility of an electron (or hole) is defined as its drift velocity in the direction of a unit electric field. Hence, the speed of collection of the charges will increase with the field strength. This is true as long as the conditions are such that negligible trapping takes place. If, however, there is trapping, the value of the mobility (as obtained from measurements of the collection time) will depend very largely on the time spent by the charges in traps before being released to continue their journey towards the electrodes. The measured value of the collection time can then be described as: (time spent in the conduction band) plus (time spent in traps); that is,

$$t_m = t_f + t_t, \text{ say,}$$

or,

$$x/k_m F = x/kF + t_t,$$

where x is the distance from the region of initial creation of the free charge to the collecting electrode, and k_m is the measured (i.e., apparent) value of the mobility. If $t_t \ll t_f$, the measured mobility will correspond approximately to the true mobility of the conduction-band electron. If $t_t \gg t_f$, the measured mobility may be very small and consequently the collection time can become very long; it may take even seconds. It is clear that erroneous results may easily be obtained by unsuitable time constants of the amplifier; the pulse may be blotted out altogether if the time constants are too short. Similar remarks will apply to the collection of holes and consequently the total collection time will correspond to the time taken to collect the slower moving charges, whether they be the electrons or the holes. In most counting crystals, it appears that the measured mobilities are relatively high and time constants as short as 10^{-5} or 10^{-6} sec can sometimes be used without fear of distorting the pulse shape. In one diamond specimen, Pearlstein and Sutton[30] have found the mobilities of electrons and holes to be 3900$\pm$15 percent and 4800$\pm$20 percent cm^2/sec volt, respectively. The reciprocal variation of the rise time with field strength has been verified by Yamakawa in AgBr[10] showing that with the time constants used, t_t is not important. Hence, solid conduction counters can have fast response times. Such characteristics are necessary in connection with high counting rates, coincidence counting, or in the resolution of events occurring at very short time intervals, for example, decay processes of nuclei. For instance, the decay of a μ-meson has been resolved by Voorhies and Street.[31] Returning to a reconsideration of the collection time it is seen that the time spent in traps plays an important part. It would appear that the response time of some counters could be decreased by reducing t_t, perhaps by using heat or infrared illumination to eject the electrons from their traps more rapidly. However, a competing process may occur since, theoretically, the mean-free path of electrons may decrease as the temperature increases. It is difficult to predict exactly what the effect of temperature will be and it would certainly be interesting to carry out such experiments.

If we assume that the number of free electrons and holes created is proportional to the energy lost by the ionizing radiation inside the crystal, so also will be the pulse height—that is, the recorded response is directly proportional to the incident energy if the latter is given up entirely

[30] E. A. Pearlstein and R. B. Sutton, Phys. Rev. 79, 907 (1950).
[31] H. G. Voorhies and J. C. Street, Phys. Rev. 76, 1100 (1949).

to the crystal. Such predictions have been verified to within experimental error in various crystals, notably AgCl[2] and diamond.[17] However, though most pulses caused by a given energy are of about the same height there is often much "straggling" in their distribution, especially for α-particle counting. Van Heerden has successfully explained the recorded pulse-height distribution in the case of AgCl though there are often many unknown factors such as the variation in sensitivity over the surface of the crystal. The property of proportional response is very useful in nuclear experiments.

In a counter behaving in a normal manner, namely, free from such complications as electron multiplication or semiconductor action, the lower energy limit to the sensitivity is determined by the width of the energy gap, of the order of a few electron-volts. It is found that the mean energy required to create an electron-hole pair is usually up to twice the width of the energy gap as found from optical absorption measurements. In gases, the energy required to create an ion-pair is of the order of 30 ev and consequently, solid counters can be expected to be the more sensitive by a factor of from 5 to 10 for a given incident energy. This means that with the present electronic techniques, the minimum detectable energy is of the order of a few kilovolts. The Geiger counter can do much better than this.

There are very few reports describing the efficiency of the response of the solid counter though it has been noted that the counting rate approaches a saturation value as the field strength is increased. For diamond, Ahearn has given a figure of 60 percent efficiency for response to α-particles though the efficiency can vary considerably over different parts of the same crystal; "maps" showing this variation have been made by Ahearn. However, for γ-rays, the efficiency of the solid counter can be up to a hundred times greater than that of the Geiger counter.

Summarizing, the useful properties of a solid conduction counter can be listed: (i) its high stopping power makes it useful for detecting radiation of high penetrating power and for use in a small volume, (ii) its fast response speed makes it suitable for high counting rates and the resolution of nuclear events at intervals greater than about a microsecond, (iii) the output pulse height is proportional to the incident energy, (iv) its sensitivity and efficiency are relatively high, especially for γ-rays, (v) it needs no "window". Finally, the counter has been found to be suitable for the detection of single α- and β-particles, γ- and x-photons, and neutrons. However, all these advantages which make the counter potentially more useful than gas-filled detectors are shared by the scintillation crystal counter. In recent years the latter has become a standard detector for use in nuclear experiments while the conduction counter is still beset with certain disadvantages and practical limitations. Foremost amongst these is the formation of a space-charge field in the crystal as the bombardment by ionizing radiations proceeds. other disadvantages are: (i) only a few crystals can be used at room temperature, (ii) variations occur in counting efficiency from crystal to crystal and even in different parts of the same crystal, (iii) the nature of the surface of the crystal greatly affects the sensitivity to low energy radiations.

ATTEMPTS TO OVERCOME THE SPACE CHARGE DIFFICULTY

When electrons and holes migrate through the crystal some of them will become trapped before reaching the electrodes. These trapped charges will set up a space-charge field in opposition to the applied field which, as the bombardment proceeds, will grow larger until the resultant field in the crystal is reduced to a very low value. The formation of the space charge manifests itself in a steady falling off of the pulse heights (and consequently, the counting rate) caused by the bombarding particles. This space-charge problem has been encountered repeatedly in photoconductivity studies in crystals and it is well known that often the space charge may be removed by warming the crystal or illuminating it with infrared light in the absence of an electric field. Presumably, the action of heat or light is to eject the electrons and holes from their traps so that they are again free to wander through the crystal lattice. In doing so, some of the electrons and holes may recombine while the rest may intermingle and become trapped in a distribution such as to cause the net remaining

space charge field to become negligible. This procedure has been adopted by the author in the study of the diamond crystal counter;[17] field pulses and light flashes were applied alternately to the crystal in a regularly repeated cycle. During the field pulse, charge pulses caused by β-rays were detected. During the subsequent brief, intense, white light flash in absence of the field, the space charge was dispersed and the whole cycle could be repeated indefinitely, the counting rate remaining perfectly steady.

From elementary pictures of the action of the counter it would appear safe to illuminate some types of crystal permanently with infrared light simultaneously with the field. Any electrons and holes that become trapped should then be quickly released to continue their journey towards their respective electrodes, thus keeping the crystal free from space charge. Experiments along these lines with diamond crystals have been made by the author,[15] Willardson and Danielson,[16] and Freeman and van der Velden.[32] Both α- and β-, as well as γ-rays have been used. It has been found that the counting rate can be made to approach a steady value in this manner though curves of counting rate against time show very peculiar behavior before the steady state is reached. The observed variations have not been explained in detail as many unknown factors can come into play. For instance, there is possibly more than one trap depth both for the electrons and the holes. Also there may be processes taking place similar to those that occur in the anomalous CdS crystals. However, such experiments may eventually lead to a practical conduction counter.

Another method that has been used to overcome the space-charge difficulty is to apply alternating fields to the crystal. This method has been used by Wouters and Christian[33] for AgCl crystals and especially by McKay for diamond.[4] During the first half-cycle of the field, a space-charge field is set up and during the second half-cycle with the field reversed, an opposing space charge field will grow. McKay used a 60-cycle sinusoidal field and pulsed a high intensity, low energy electron beam onto the crystal for a few microseconds at both the crests and the troughs of the field. By slight adjustment of the relative lengths of the beam pulses at these two parts of the cycle, McKay showed that the crystal could be made virtually free from space charge at the end of each cycle since, if the field was then removed, no space charge bombardment current could be detected to within the sensitivity of his apparatus.

All the above methods obviously limit the usefulness and ease of operation of a solid conduction counter; consequently, it is relevant to point out that sometimes the counter can be operated in the normal manner with a steady applied field and in the dark in such a way as to minimize, though not remove, the effect of the space-charge field. Earlier, it was described how the curve of the pulse height against applied field showed a pronounced saturation when the mean free paths of the free electrons and holes became long compared with the dimensions of the crystal, a situation that occurs at high field strengths. Hence, using high applied fields, quite a large space-charge can form before there is any appreciable reduction in the pulse height and in this way quite a large number of particles can be counted before the crystal has to be "rejuvenated."

CONCLUSIONS

From this discussion of solid conduction counters it will be seen that they cannot take their place yet as standard tools in nuclear physics largly on account of the formation of space-charge. As further developments take place this situation may become altered; suitable crystal substances having a trap density several orders of magnitude lower than those in use today may be found and these could be expected to show a much longer life. However, solid counter studies have shown a new way to investigate the crystalline state. In particular, it has been indicated how the processes involved in the trapping and detrapping of holes and electrons and allied phenomena can be studied. Estimates can be made of trap densities, trap depths, and trapping and detrapping probabilities. Further, the theories of the behavior of free electrons and holes in crystal lattices can be checked to some extent.

[32] H. A. van der Velden and G. P. Freeman, Physica 16, 493 (1950).

[33] L. F. Wouters and R. S. Christian, Phys. Rev. 72, 1127 (1947).

The Peripheral Potential Drop of an EMF Inside A Conducting Medium

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(Received November 9, 1951)

When an emf, embedded in an electrically conducting medium, is measured by peripherally applied probe electrodes, the ratio of the measured potential drop to the total emf must be known. It is shown how this ratio can be obtained by computation. The specific case of an electromagnetic blood flow meter is used as the basis of computation.

IT is sometimes necessary to measure the magnitude of an emf located inside a body of essentially uniform electric conductivity. Such measurement can be effected by means of two probe electrodes applied to two suitable points of the periphery of the body, considered as two-dimensional in the following.

One of the chief questions that arise in this connection is what fraction of the total emf E is measurable, if the probe electrodes are located some distance away from the source and separated from it by the electrically conducting medium. The latter acts as a relatively low parallel resistance across the emf, and since only a certain fraction of that resistance is accessible, it is important to know the ratio of the measurable potential drop to the total emf. This ratio determines the sensitivity of the measuring instrument required for the purpose and also permits immediate calculation of the total emf from the measured potential drop. It is the solution of this particular problem with which this paper is concerned.

The solution of two-dimensional electric flow and potential patterns in conducting bodies can be solved experimentally by the tank method or by computation. In the present study the computational method was used. The computation employed involved several approximations, but was considered sufficiently accurate for the purpose.

THE ELECTROMAGNETIC METHOD

In order to be specific, a particular case was considered. It may be looked upon as an illustration of a computational method of general applicability. The particular case under consideration is the measurement of blood flow rate by means of the electromagnetic method.

The electromagnetic method of measuring flow rate in a tube consists of applying a magnetic field normal to the tube axis and a pair of probe electrodes normal to both the magnetic field and tube axis. Owing to induction, an emf E is produced between the electrodes; its magnitude in volts is

$$E = 10^{-8} Hvl, \tag{1}$$

where H is the magnetic field in gauss, v the linear flow rate in cm/sec, and l the inside diameter of the tube in cm. This principle has found application both in engineering[1,2] and in medicine.[3] Concerning this latter, the method requires isolation of the blood vessel to be investigated.

This latter circumstance is an obvious drawback, and a technique, permitting nonsurgical application of the principle, would have definite advantages. A technique of this kind would involve the application of both the magnetic pole pieces and the proble electrodes to the periphery of an extremity in which the venous flow is stopped by a tourniquet but the arterial flow is not. This latter is necessary; otherwise, emf's from the arterial and venous flow would cancel each other. For about a minute from the application of the tourniquet the arterial flow rate due to the expansibility of the capillaries

[1] J. S. Arnold, Rev. Sci. Instr. 22, 43 (1951).
[2] A. J. Morris, and J. H. Chadwick, Transactions Am. Inst. Elec. Engrs. 70, 346 (1951).
[3] K. E. Jochim, Methods in Medical Research 1, 108 (1948).

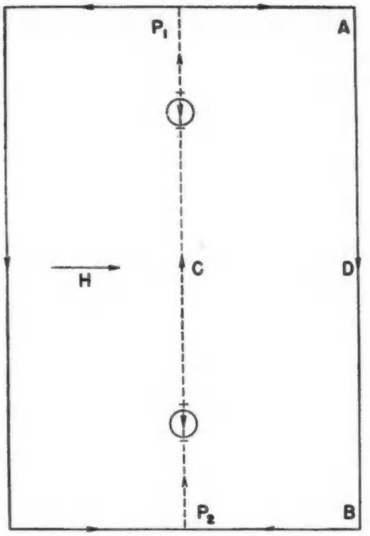

FIG. 1. Schematic cross section of conducting body, as used for computation.

remains essentially unaffected and this time should be sufficient to obtain the required information.

For the following computation, the wrist was considered; Fig. 1 shows an equivalent rectangle as its cross section. The size of the two arteries and their location along the vertical axis are to scale; their location along the horizontal axis was, for sake of simplicity, assumed to be symmetrical. With the direction of H as shown, the emf is oriented along P_1P_2, the two points P_1 and P_2 showing the positions of the probe electrodes. In case the arterial cross section is small compared with that of the surrounding tissue, it is possible to substitute for each induced emf a source and a sink as shown. The calculated field pattern deviates from the true field only in the immediate vicinity of the arteries; at any other location the two fields are similar.

FLOW LINES

Concerning the flow lines, we know that because of symmetry of the field two short lines connect the positive poles with the negative ones; another group of lines go in the opposite direction along the entire center line and the periphery as shown. Since the flow lines obey Laplace's equation, one can consider the pattern in the rectangle P_1P_2AB as a boundary-value problem and develop the flow function v as a Fourier series.[4] The side P_2B is taken as the x axis with P_2 at $x=0$ and B at $x=a/2$. The side P_2P_1 is taken as the y axis with P_2 at $y=0$ and P_1 at $y=b$. The small separation of the source and sink is called c and the section of b above the top source is called g. The value of v over the boundary is taken as zero, except over the two short sections c, where it is taken as unity. We then have for the flow function

$$v=\sum_m[B\exp(-m\pi x/b)+N\exp(m\pi x/b)]\times\sin(m\pi y/b), \quad (2)$$

where m represents all odd positive integers starting from 1, and B and N are functions of m. The sum S of B and N is then given by

$$S=(8/m\pi)\sin(m\pi c/2b)\sin m\pi[(2g+c)/2b]. \quad (3)$$

The constants B and N can be calculated from the boundary conditions. For the numerical data as taken below, it may be shown that for $m\geqq 3$, B is large compared to N. One obtains approximately

$$B=S \quad (4)$$

and

$$N=-S\exp(-m\pi a/b). \quad (5)$$

For $m=1$, one has for $a/b=0.667$ (see below)

$$B=1.14S,$$

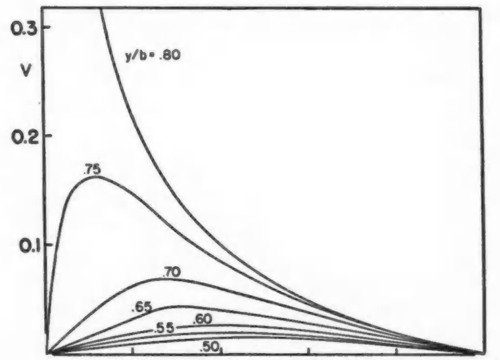

FIG. 2. Flow function v as function of plane coordinates x and y (the latter as parameter, from 0.80 down).

[4] L. A. Pipes, *Applied Mathematics for Engineers and Physicists* (McGraw-Hill Book Company, Inc., New York, 1946), p. 405.

and

$$N = -0.141S.$$

It is now possible to evaluate the electric flow pattern. In order to do this, numerical values must be taken. The following set of values is in keeping with reality: $a=4$ cm, $b=6$ cm, $c=0.3$ cm, and $g=1.05$ cm. The pattern was evaluated for the rectangle P_1ACD, since the other three are the same. First v was calculated as a function of $2x/a$, the ordinate y/b being the parameter. The family of curves so obtained is given in Figs. 2 and 3. By suitable transposition of these data, the flow pattern is obtained; Fig. 4 shows it for the rectangle P_1ACD. The curves are lines $v=\text{const}$; the values of v are shown. The figure indicates one-quarter of the wrist cross section, the semicircle to the left representing one of the arteries.

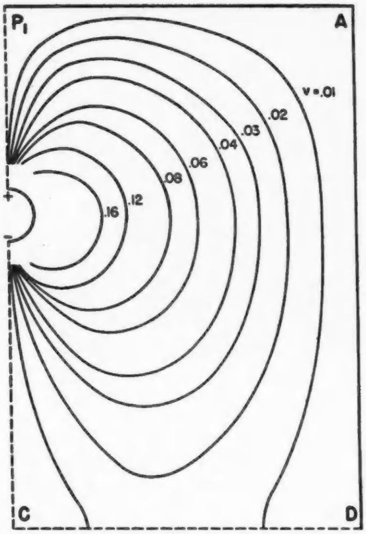

FIG. 4. Flow pattern in cross section of conducting body around one of the emf's.

THE POTENTIAL FUNCTION

The crowding of the lines near the center line P_1P_2 shows qualitatively that the potential drop along the circumference P_1ABP_2, which is measured by the probes, is relatively small. This potential drop can now be evaluated quantitatively. Let us denote the potential function by u and the direction along the periphery of the rectangle P_1ABP_2 by s. If the direction normal to the periphery toward the inside of the rectangle is n, then, by Laplace's equation

$$(\partial u/\partial s) = -(\partial v/\partial n). \tag{6}$$

Hence the potential difference between any two points 1, 2 along the periphery is given by the definite integral

$$u_{12} = -\int_1^2 (\partial v/\partial n)ds. \tag{7}$$

Since $\partial v/\partial n$ is known by Eq. (2), the potential drops can be computed.

In this manner the potential drop along the center line, except the short sections c, becomes

$$u_{P(2)P(1)} = 2\sum_m (B-N)[(m\pi/4)S-1], \tag{8}$$

along each of the two sections of length $a/2$

$$u_{P(1)A} = \sum_m S[1-2\exp(-m\pi a/2b)] \tag{9}$$

and along the line AB,

$$u_{AB} = 4\sum_m S\exp(-m\pi a/2b). \tag{10}$$

The terms in Eqs. (9) and (10) are good approximations for $m \geqq 3$. For $m=1$, the term in Eq. (9) is $0.48S$, and in Eq. (10) it is $1.60S$.

What is needed is the external potential drop u_e along the line P_1ABP_2 in relation to the total drop along the entire circumference, which is identical with the generated emf E of Eq. (1). Carrying out the numerical computation, the relatively poor convergence of the series involved introduces a further approximation. With the

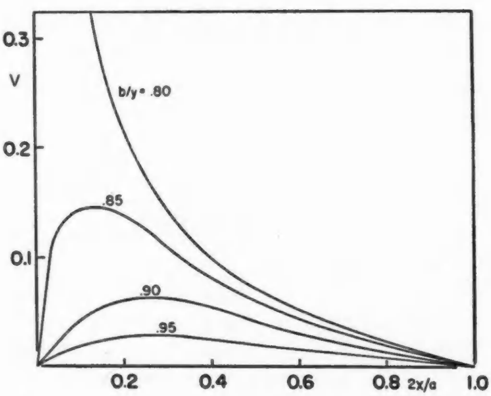

FIG. 3. Flow function for parameters from 0.80 up.

numerical data as taken, Eq. (8) yields the sum 3.00, Eq. (9) yields 0.125 (this must be taken twice), and Eq. (10) yields 0.155. The total potential difference is then 3.40, and the peripheral drop is 0.40; hence

$$u_e/E=0.12,$$

which shows that only about 12 percent of the total emf is measurable by the technique in question.

The value of E may be estimated. A magnetic field of about 3000 gauss is easily obtainable from a small electromagnet and 5-cm air gap. The length l in Eq. (1) is 0.6 cm. The linear blood flow can be estimated[5] to be about 8 cm/sec. According to these figures, E would be 150 microvolts. Equation (11) then gives for u_e, the measurable potential, 17 microvolts as the expected order of magnitude.

As pointed out earlier, a similar type of calculation can be carried out for other analogous problems the geometry of which differs from the one considered above.

[5] C. H. Best and N. B. Taylor, *Physiological Basis of Medical Practice* (Williams and Wilkins, Baltimore, Maryland, 1947), p. 149.

A Solenoidal Spectrometer for the Undergraduate Laboratory

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(Received November 26, 1951)

A solenoidal spectrometer is described. This instrument has several features which make it especially adaptable to use in an undergraduate laboratory. One is that the coil geometry is such that the student may readily verify that the range of variation of the magnetic field on the principal axis is less than ¼ percent. Secondly, the baffle geometry is of sufficient simplicity to enable the better student to calculate not only the momentum admitted by the baffle system, but as well to predict the line shape for a monoenergetic source, and thus to obtain the theoretical resolution of the instrument. Thirdly, as a result of the first-mentioned features, the instrument is absolute. Construction details are included. For the instrument described, the absolute accuracy is better than 0.4 percent, the resolution is 1.9 percent, and the useful solid angle is 0.8 percent.

SEVERAL articles concerned with the properties of solenoidal spectrometers have recently been published.[1] These articles have been concerned with instruments of optimum design which were to be used for research purposes. Accordingly the theory in each case involves the use of the "ring focus."[2] When a spectrometer was constructed at this university for use in an undergraduate laboratory, it was first assembled using the ring-focus baffle, following the theory of Du Mond.[1] A semester's use with this arrangement proved somewhat unsatisfactory for two reasons. In the first place, since the mean angle of emission was only about 14°, the ring focus baffles were of such small dimensions and had to be placed so near the detector, that their placement became very critical. Secondly, the complete theory of the instrument as given by Du Mond and others is of sufficient complexity so that most students were unwilling or unable to study it enough to

[1] Persico and Geoffrion, Rev. Sci. Instr. **21**, 945 (1950). Persico, Rev. Sci. Instr. **20**, 191 (1949). Du Mond, Rev. Sci. Instr. **20**, 160 (1949).

[2] Witcher, Phys. Rev. **60**, 32 (1941). Frankel, Phys. Rev. **73**, 804 (1948).

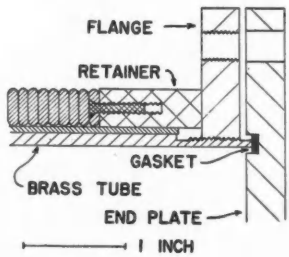

FIG. 1. Construction details at an end of the spectrometer. The main coil consists of a single layer of copper ribbon 0.1 inch by 0.38 inch wound on edge on the brass tube which constitutes the vacuum chamber.

have anything like a thorough appreciation of the operation of the spectrometer.

We therefore decided to investigate the possibilities of using the very simple baffle system which is usually sketched when one is first introducing the solenoidal spectrometer, namely, a set of "angle" selecting baffles near the source, and an "energy" selecting hole in a position just in front of the detector. A simple theory for the line shape and momentum resolution is possible for this baffle geometry. It appears to be a very suitable geometry for use in a student's instrument. Accurate experimental results are obtained, and the theory is so simple that practically every student thoroughly understands the details of the action of the baffles. The better students can, with little aid, develop the theory.

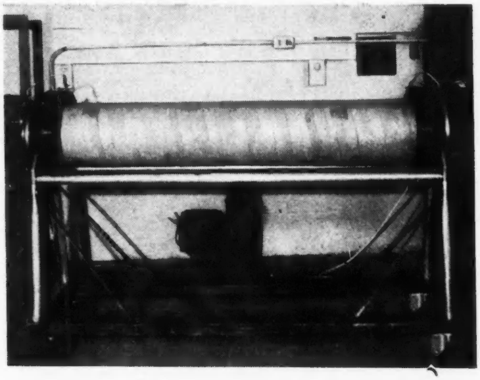

FIG. 3. A side view of the spectrometer. The cardboard cylinder serves as a duct for the cooling air. A correction coil is seen at each end of the instrument.

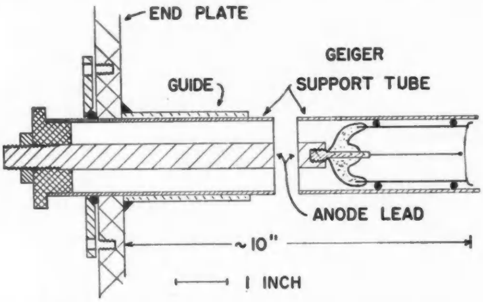

FIG. 2. Details to show the means of support of the Geiger counter. The O-ring vacuum seal between the cylindrical cathode and the support tube allows the anode lead to remain at atmospheric pressure.

CONSTRUCTION AND DIMENSIONAL DETAILS

The vacuum chamber of the spectrometer consists of a brass tube which is 63.3 inches long. It has an internal diameter of 8 inches and an outside diameter of $8\frac{1}{4}$ inches. The tube is covered with several layers of paraffined paper which serves as electrical insulation for the solenoid coil. The total thickness of this insulation is a little less than $\frac{1}{16}$ inch.

The solenoid is wound with heavy Formvar-covered copper ribbon. This ribbon is approximately 0.1 inch by 0.38 inch. It is wound *on edge*, so that there are approximately ten turns per inch. The solenoid proper consists of a single layer of such winding. The coil resistance is about $\frac{1}{2}$ ohm.

Figure 1 shows the construction details at one end of the spectrometer. The last turn on the solenoid is screwed to the Bakelite retainer. The retainer, a cylindrical piece, in turn is held in compression against the copper winding by a flange. The flange screws onto the brass tube which constitutes the vacuum chamber.

In Fig. 2 is shown the means of supporting the Amperex type 200 N Geiger tube. This tube is filled to a pressure of approximately half an atmosphere, so that it may be used at either atmospheric pressure or at high vacuum. The Geiger tube is supported by an O-ring vacuum seal as shown. This provides a convenient means of allowing the anode lead to remain at atmospheric pressure. Since the anode lead is at atmospheric pressure, no difficulty with discharging from the anode lead within the spectrometer vacuum is experienced. Requirements on the spectrometer vacuum are thus materially

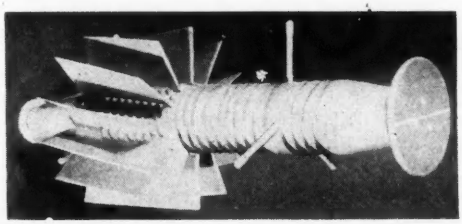

FIG. 4. A photograph of the lead absorber which is placed between the source and the detector on the principal axis. The helicoidal charge-sign selecting baffles are supported from the lead "pig." The inner (12°) angle-selecting baffle is seen at the right.

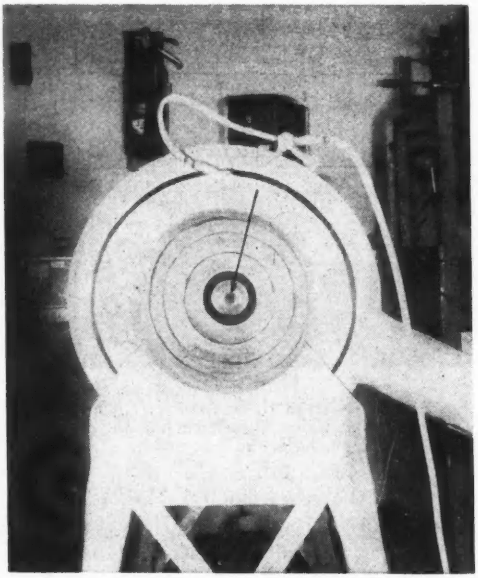

FIG. 5. A photograph, with the absorber of Fig. 4 removed, showing the detector end of the spectrometer. The small hole which is indicated by the arrow is the energy-selecting baffle.

reduced. It is apparent from Fig. 2 that the anode lead also serves as a longitudinal support for the Geiger tube. The entire Geiger support tube is adjustable longitudinally ± 1 inch about its nominal proper location.

Cooling of the solenoid coil is provided by forced-air circulation. A length of cardboard tubing whose internal diameter is one-half an inch greater than the outside diameter of the copper winding confines the air to a high velocity flow along the surface of the coil. The air inlet is at the center of the instrument and from there the flow is toward each end. A $\frac{1}{2}$-horsepower, 3450-rpm motor driving a centrifugal blower furnishes a pressure head of 6 inches of water at the air inlet to the spectrometer. When the spectrometer is operated at 80 amperes, 40 volts, and 3.2 kilowatts input, a coil temperature rise to about 65°C results. Since 80 amperes focuses particles with an energy of 1.7 Mev, this is about the maximum energy at which the instrument may be held in continuous operation. Operation at this power requires the use of dry ice in a hopper at the blower intake.

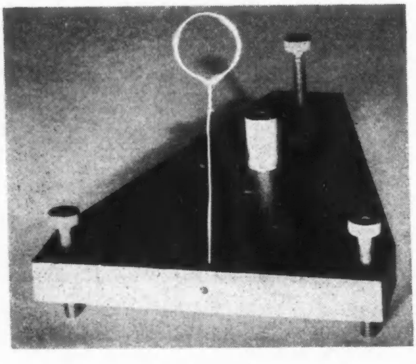

FIG. 6. A photograph showing the means of mounting and adjusting the source.

Figure 3 is a photograph showing a side view of the spectrometer. Note the motor and blower, the cardboard air duct, the correction coils, and aluminum frame which is mounted on casters.

Figure 4 is a photograph of the lead absorber which is placed on the axis of the spectrometer between the source and detector. The lead itself is three inches in diameter and twenty-one inches long. A set of helicoidal baffles is mounted near the detector end of the absorber. These of course are used to limit the transmission for a given direction of the magnetic field to particles of a single sign. Near the source end of the absorber is seen the inner (12°) angle-selecting baffle. This baffle is accurately centered within the vacuum chamber by adjustment of the length of the six supporting legs which carry the lead absorber.

TABLE I. Coil dimensions.

	Main coil	Correction coils
Inside radius (cm)	10.60	15.22
Outside radius (cm)	11.57	16.19
Mean radius (cm)	11.09	15.71
Length (cm)	150.6	5.1
Turns per meter	403.3	392

The outer (16°) angle-defining baffle consists of a cylindrical ring placed at the proper axial distance from the source. It is automatically centered on installation, for the outside diameter of the ring is a snug fit to the inside diameter of the vacuum chamber. There are other inner and outer baffles, but these act neither as energy nor

angle-selecting baffles—rather as baffles whose purpose is to reduce the scattered background.

Figure 5 is a photograph showing the detector end of the spectrometer. The arrow indicates the energy-selecting baffle. This baffle is supported by a large lead casting whose minimum wall thickness is $2\frac{1}{2}$ inches. This lead casting serves to reduce the background due to cosmic rays and radioactive sources which are nearby. The energy-selecting baffle is accurately centered to the axis of the vacuum chamber by adjustment of the length of the supporting legs of the above lead casting.

Figure 6 is a photograph of the source holder. The length of the three legs is adjustable, so the source, which mounts on the wire loop, may be accurately adjusted to the axis of the vacuum chamber.

TABLE II. Magnetic field contributions. Distance from center of the solenoid in cm.

	0	10	20	30	40	50	55
Main coil	1.9786	1.9776	1.9738	1.9658	1.9494	1.9120	1.8740
Nearby correction coil	0.0032	0.0050	0.0081	0.0146	0.0293	0.0677	0.1081
Remote correction coil	0.0032	0.0021	0.0015	0.0011	0.0008	0.0006	0.0005
Total	1.9850	1.9847	1.9834	1.9815	1.9795	1.9803	1.9826

THE MAGNETIC FIELD

The dimensions of the main coil and of the correction coils are given in Table I. The 5.1-cm length of the correction coils is located in the region between 1.3 cm and 6.4 cm from the ends of the main coil. This coil configuration produces a magnetic field which, on the axis of the solenoid, is uniform to within $\frac{1}{4}$ percent over more than 70 percent of the length of the main solenoid winding. Table II shows the results of a calculation of the field on the principal axis at various distances from the center of the solenoid. We have tabulated the value of the terms in parentheses in the expression $H=(2\pi/10)NI\ (\cos\theta_1 - \cos\theta_2)$, in which H is the field strength in oersteds, N is the number of turns per cm length, I the magnet current in amperes, and θ is the angle between the axis of the solenoid and the end of the winding for the point in question. The mean coil radii were used for the computations.

It is seen that the field variation is $\frac{1}{4}$ percent on the axis of the solenoid in the region which is used. Calculations for both the main coil and for the correction coils show that an error of $\frac{1}{3}$ percent is incurred if one uses either the inner or the outer coil radii for the above computations. The mean values of the calculations for the outer and inner coil radii of course differ by negligible amounts from the values shown in Table II.

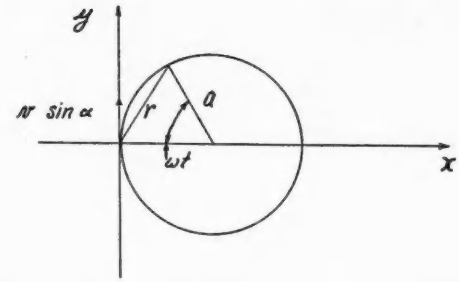

FIG. 7. End view of the geometry of the starting conditions.

In evaluating the value of H to be used, we have chosen a value from Table II of 1.984, weighting values near the center rather heavily since the curvature of the particle is greatest near the central part of the trajectory. We obtain H (oersteds) $=B$ (gauss) $=5.027\ I$ (amp).

This is the value of the field on the principal axis. We have experimentally determined that at no point on the particle trajectory does the field differ by more than $\frac{1}{4}$ percent from the value at the center of solenoid. This may also be established from the numerical calculations of the field on the principal axis, following Siegbahn,[3] from the equation

$$H_z(z,r)=H(z,0)-\frac{r^2}{4}\frac{\partial^2 H_z}{\partial z^2}+\cdots.$$

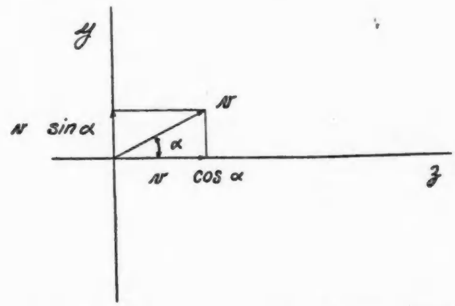

FIG. 8. Side view of the geometry of the starting conditions.

[3] Kai Siegbahn, Arkiv. Mat. Astron. Fysik 30, 1 (1944).

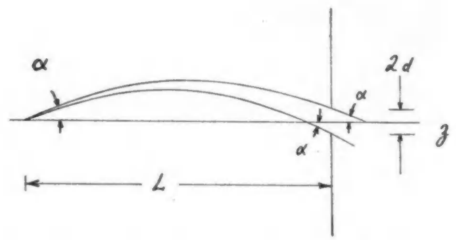

FIG. 9. Geometrical situation for determining the range of momenta transmitted by the "energy-selecting" hole for a given angle of emission.

The use of this equation to estimate the field for $r=4$ inches and in the plane of symmetry results in a deviation of less than 0.1 percent from the field on the principal axis.

THEORY

The z-axis is taken to be the axis of symmetry of the instrument. The plane perpendicular to the z-axis is the xy plane. A source is placed at $x=y=z=0$. Consider a particle of mass m and velocity v which is emitted at $t=0$ at an angle α to the z axis and in the yz plane. Figures 7 and 8 portray the situation. The projection of the trajectory in the xy plane will of course be a circle. Let its radius be a. Then we have

$$x=a(1-\cos\omega t) \tag{1}$$

$$y=a\sin\omega t \tag{2}$$

$$z=(v\cos\alpha)t. \tag{3}$$

Now $\omega=Be/m$ and $a=p\sin\alpha/Be$, where p is the particle momentum, B the strength of the magnetic induction in gauss, m the mass in grams and e the charge on the particle in electromagnetic units. When we solve for r, the displacement of the particle perpendicular to the z axis, we obtain

$$r=(x^2+y^2)^{\frac{1}{2}}=\frac{2p}{Be}\sin\alpha\sin\left(\frac{Bez}{2p\cos\alpha}\right) \tag{4}$$

or

$$r=D\sin\alpha\sin\left(\frac{z}{D\cos\alpha}\right), \tag{5}$$

where $D=2p/Be$. This is the familiar result that the path of the particle in a plane which rotates about the z axis at a rate such that it always contains the particle is sinusoidal.

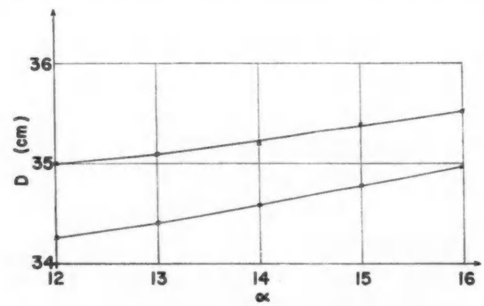

FIG. 10. A plot of the range of the values of $D=2p/Be$ which are transmitted by the spectrometer geometry *vs* the angle of emission α of the particle with respect to the principal axis of the spectrometer.

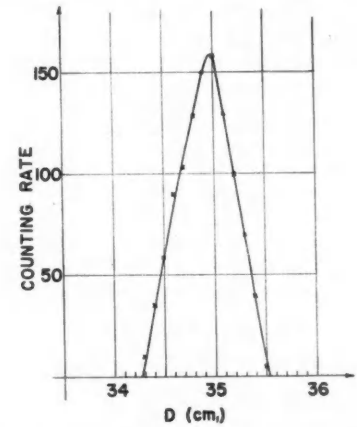

FIG. 11. Theoretical line shape for a monoenergetic point source, as determined from the curves of Fig. 10. The predicted full width at half maximum intensity $\Delta D/D=\Delta p/p=1.8$ percent.

Consider Fig. 9. Two particles which are emitted at an angle α with the z axis are shown. We wish to calculate the range of D values which will be allowed to pass through the opening of diameter $2d$ which constitutes the energy-selecting slit in the geometry which is used in this spectrometer. Calling D^+ the largest and D^- the smallest values of D which will be transmitted, we have, for the first crossing of the z axis

$$L+d\,\mathrm{ctn}\alpha=\pi D^+\cos\alpha$$

$$L-d\,\mathrm{ctn}\alpha=\pi D^-\cos\alpha.$$

These equations result from the requirement that the argument of the sine term in Eq. (5) must equal π for the first crossing of the z axis. Solving

we obtain

$$D^{\pm} = \frac{L \tan\alpha \pm d}{\pi \sin\alpha}.$$

The geometry of this spectrometer consists of a set of *angle-selecting baffles* placed near the source. These allow particles which are emitted in the angular range 12°–16° to pass on down to the energy (momentum) selecting hole.

Figure 10 shows a plot of the range of D values which will be transmitted by the instrument for angles in the range 12°–16°. For our instrument $L=106.4$ cm and $d=0.24$ cm.

Consider now a monoenergetic point source. As we vary the current through the solenoid, the valve of B changes, and thus the value of D for the particles being emitted by the source varies.

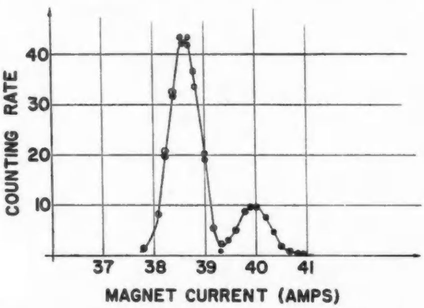

FIG. 12. Experimental line shape for a Cs^{137} source. The measured resolution is <2 percent.

The curves of Fig. 10 will determine the useful angular range of the instrument for each value of D. For example, only for a $D=35.0$ cm will particles in the entire range 12°–16° be able to pass from the source to the detector. We thus determine the curve of Fig. 11 which shows the counting rate *vs* D for our instrument.

We note that the predicted momentum resolution $\Delta p/p=\Delta D/D=1.8$ percent. Also, the maximum counting rate occurs at $D=35.0$ cm; the high energy side of the curve intercepts the D axis at $D=35.5$ cm; and the center of gravity of the line is at $D=34.9$ cm.

For operation at the peak of the counting rate curve, the angular range 12°–16° which is in use means that about 0.85 percent of all the particles emitted by an isotropic monoenergetic source will enter a counter which is placed behind the energy selecting hole.

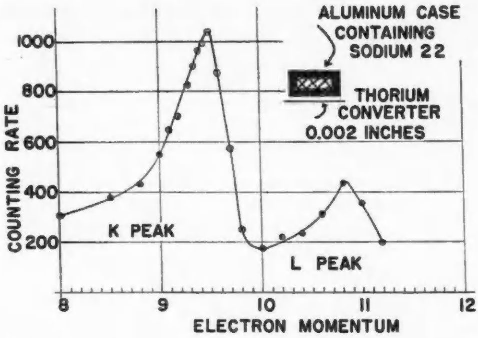

FIG. 13. Experimental results obtained in an experiment performed in the undergraduate laboratory. The energy of the annihilation radiation from the positrons of Na^{22} is measured, using a thorium converter. The error in the absolute measurement is less than ½ percent.

EXPERIMENTAL RESULTS

Figure 12 shows the results of a run made with a Cs^{137} source. The experimental resolving power is 1.9 percent as compared with the theoretically predicted value of 1.8 percent. The measured momentum of the line is 3390 gauss cm as compared with the published value of 3381 gauss cm. This difference of ⅓ percent in the momentum is consistent with the precision which the spectrometer is expected to attain. The magnetic field is only uniform to about ¼ percent, and then there is about a ⅓ percent error possible in determining the position of the peak of the counting rate curve.

Figure 13 shows the results of an experiment which is performed in the undergraduate laboratory in which the instrument is used. After establishing a plateau for the Geiger counter, the energy of the annihilation radiation from the positrons of Na^{22} is measured, using a thorium foil converter. The final error in the energy of the annihilation quanta is less than ½ percent.

It is a pleasure to thank Mr. Howard Keller and Mr. Alfred Mann for their aid in the design and testing of the spectrometer. Our thanks are also due Mr. E. Griffiths, who is in charge of the Physics Department machine shop, for his many suggestions regarding construction details and for supervising the construction of the spectrometer. Mr. Robert Nordberg was responsible for the electrical installation.

Conditions for the Derivation of the Stress Deviator Tensor

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(Received December 10, 1951)

The decomposition of the stress tensor into the spherical stress tensor and the stress deviator found in theories of plasticity is shown to depend upon the postulate that the stress tensor is the tensor sum of two physically independent stress constituents. The decomposition is derived by two different methods, the first method based on energy arguments, and the second method based on cause and effect arguments.

INTRODUCTION

FROM the onset, it will be best to state the purpose of this paper which is to define the *conditions* on which the stress (strain) tensors can be decomposed into the two stress (strain) tensors that are commonly called[1] the spherical stress tensor and the stress deviator. The actual decomposition of stress is well known, and is arbitrarily assumed as a mathematical convenience in current theories of plasticity. While this assumption is justified by the results so obtained, no consideration is given to the physical requirements, i.e., *conditions*, of such a decomposition. The properties of the resulting tensors, hereafter called stress (strain) constituents, have been investigated by a great many investigators concerned with theories of failure of bodies.[2] This paper will investigate the principles underlying the decomposition itself.

By way of review, the decomposition of stress and its resulting implications will be presented after the fashion that various Russian authors introduce it in their mathematical theories of plasticity.[3] The following concepts are used in characterizing the state of stress under which a material enters the plastic state:

(1) The state of stress at a generic point is determined by the six components of the stress tensor, written in engineering symbols as

$$S=\begin{pmatrix}\sigma_x & \tau_{yx} & \tau_{zx}\\ \tau_{xy} & \sigma_y & \tau_{zy}\\ \tau_{xz} & \tau_{yz} & \sigma_z\end{pmatrix}.$$

(2) The mean normal stress is defined as

$$\sigma=\tfrac{1}{3}(\sigma_x+\sigma_y+\sigma_z).$$

(3) The stress is decomposed into the dilatational stress constituent (called the spherical stress tensor)

$$S_1=\begin{pmatrix}\sigma & 0 & 0\\ 0 & \sigma & 0\\ 0 & 0 & \sigma\end{pmatrix},$$

which corresponds to a change of volume only, and

(4) The distorting stress constituent (called the stress deviator)

$$S_2=\begin{pmatrix}\sigma_x-\sigma & \tau_{yx} & \tau_{zx}\\ \tau_{xy} & \sigma_y-\sigma & \tau_{zy}\\ \tau_{xz} & \tau_{yz} & \sigma_z-\sigma\end{pmatrix}=\begin{pmatrix}\sigma_x' & \tau_{yx} & \tau_{zx}\\ \tau_{xy} & \sigma_y' & \tau_{zy}\\ \tau_{xz} & \tau_{yz} & \sigma_z'\end{pmatrix},$$

which corresponds to a change of shape only.

(5) The first and second invariants of S_2 define the "conditions of plasticity,"[4]

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[1] W. Prager and P. G. Hodge, *Theory of Perfectly Plastic Bodies* (John Wiley and Sons, Inc., New York, 1951), p. 15.

[2] R. Hill, *The Mathematical Theory of Plasticity* (Oxford University Press, New York, 1950), pp. 15–19.

[3] Kachanov, Beliaev, Ilyushin, Mostow, and Gleyzal, *Plastic Deformation Principles and Theories* (Mapleton House, Brooklyn, New York, 1948), 192 pp.

[4] A. Nadai, *Theory of Flow and Fracture of Solids* (McGraw-Hill Book Company, Inc., New York, 1950), Chp. 27; Hill, reference 2, pp. 19–23; Prager, reference 1, pp. 21–25. The "conditions of plasticity" need not necessarily be derived in the above fashion. The origin of these equations properly belonged to St. Venant, Hencky, Huber, v. Mises, and others who were interested in finding the limiting case where Hooke's law ceased to be valid. Therefore, these equations are often called yield criteria. We are also indebted to A. Nadai for his geometrical interpretation of the above equations (pp. 96–106). An investigation of the actual "conditions of plasticity" is not the purpose of this paper, however, and they are only presented to demonstrate the importance of an examination of the above decomposition of stress to mathematical theories of plasticity.

1. $\sigma_x'+\sigma_y'+\sigma_z'=0$
2. $(\sigma_x-\sigma_y)^2+(\sigma_y-\sigma_z)^2+(\sigma_x-\sigma_z)^2$
$+6(\tau_{xy}^2+\tau_{yz}^2+\tau_{zz}^2)$
$=$constant during flow.

(6) Corresponding operations and equations pertain to the strain.

Although the decomposition $S=S_1+S_2$ is taken as axiomatic in mechanical theories of plasticity, this is somewhat unsatisfactory from a strictly fundamental point of view. The question arises, are there other ways of decomposing S so that one of its constituents produces the desired yield criteria? If this decomposition is unique, what physical meaning is implied by the uniqueness itself? What is implied, in terms of elastic theory, by making this decomposition unique?

The decomposition, $S=S_1+S_2$, will be derived from two different viewpoints. The first derivation is based on the premise that if S_1 and S_2 are quantities representing different physical entities, the elastic strain energy of each constituent is independent and the total energy of strain is therefore the sum of the two energies of the constituents. This point of view is attractive to those who find energy arguments appealing. The second derivation is based upon a causal argument. If the stress constituents, S_1 and S_2, are different and fundamental physical entities, the relation between them and the corresponding strain constituents, $\sum_1$ and $\sum_2$, is one of strict proportionality. This point of view is attractive to those who find cause and effect arguments appealing.

NOTATIONS AND FUNDAMENTAL DEFINITIONS

In all subsequent arguments, the stress and strain notations of Sommerfeld[5] will be used since this notation is more adapted to the summation technique of analysis. The following table is useful in comparing the notation of Sommerfeld with the standard engineering usage.

Sommerfeld	ϵ_{xx}	ϵ_{yy}	ϵ_{zz}	$\epsilon_{xy}=\epsilon_{yx}$	$\epsilon_{yz}=\epsilon_{zy}$	$\epsilon_{zx}=\epsilon_{xz}$
Engineering usage	ϵ_x	ϵ_y	ϵ_z	$\frac{1}{2}\gamma_{xy}=\frac{1}{2}\gamma_{yx}$	$\frac{1}{2}\gamma_{yz}=\frac{1}{2}\gamma_{zy}$	$\frac{1}{2}\gamma_{zx}=\frac{1}{2}\gamma_{xz}$
Sommerfeld	σ_{xx}	σ_{yy}	σ_{zz}	$\sigma_{xy}=\sigma_{yx}$	$\sigma_{yz}=\sigma_{zy}$	$\sigma_{zx}=\sigma_{xz}$
Engineering usage	σ_x	σ_y	σ_z	$\tau_{xy}=\tau_{yx}$	$\tau_{yz}=\tau_{zy}$	$\tau_{zx}=\tau_{xz}$.

In order that a state of stress at a point does not depend on the arbitrary choice of a set of axes of reference, three different relations between the stress components must hold. These relations are called the invariants under rotation. Only the first two invariants of stress and strain will be needed subsequently. They are

$$I_1(\epsilon)=\epsilon_{xx}+\epsilon_{yy}+\epsilon_{zz}=\text{constant} \tag{1}$$

$$I_2(\epsilon)=\epsilon_{xy}^2+\epsilon_{yz}^2+\epsilon_{zx}^2-\epsilon_{xx}\epsilon_{yy}-\epsilon_{xx}\epsilon_{zz}-\epsilon_{yy}\epsilon_{zz}=\text{constant} \tag{2}$$

$$I_1(\sigma)=\sigma_{xx}+\sigma_{yy}+\sigma_{zz}=\text{constant} \tag{3}$$

$$I_2(\sigma)=\sigma_{xy}^2+\cdots=\text{constant}. \tag{4}$$

For convenience, we shall label

$$I_1(\epsilon)=e$$
$$I_1(\sigma)=\theta.$$

The strain-stress relationships of isotropic bodies, where strain is in terms of stress, are

$$\epsilon_{xx}=\frac{1+\nu}{E}\sigma_{xx}-\nu(\sigma_{xx}+\sigma_{yy}+\sigma_{zz})$$

$$\epsilon_{xy}=\frac{1+\nu}{E}\sigma_{xy},$$

where ν and E are Poisson's ratio and Young's modulus, respectively. The above equations are more adapted to the mathematical operations to follow if they are written in condensed form with the Kronecker delta

$$\epsilon_{ik}=\frac{1+\nu}{E}\sigma_{ik}-\nu\delta_{ik}\theta, \tag{5}$$

where i and k may take on values of x, y, or z, and as usual,

$$\delta_{ik}=1,\quad i=k$$
$$=0,\quad i\neq k.$$

[5] A. Sommerfeld, *Mechanics of Deformable Bodies* (Academic Press, Inc., New York, 1950), pp. 9 and 61.

The stress-strain relationships of isotropic bodies, where stress is in terms of strain, are

$$\sigma_{xx}=2\mu\epsilon_{xx}+\lambda(\epsilon_{xx}+\epsilon_{yy}+\epsilon_{zz}).$$
$$\sigma_{xy}=2\mu\sigma_{xy},$$

where 2μ and λ, introduced by Lamé, are known as the Lamé moduli.

In condensed writing, the above is written

$$\sigma_{ik}=2\mu\epsilon_{ik}+\lambda\delta_{ik}e. \tag{6}$$

The mean normal stress is

$$\sigma_m=\tfrac{1}{3}(\sigma_{xx}+\sigma_{yy}+\sigma_{zz})=\tfrac{1}{3}\theta, \tag{7}$$

and the mean normal strain is

$$\epsilon_m=\tfrac{1}{3}(\epsilon_{xx}+\epsilon_{yy}+\epsilon_{zz})=\tfrac{1}{3}e. \tag{8}$$

The elastic strain energy of a body that has undergone deformation is usually defined[6] with geometric arguments. For example, it is usually demonstrated that the work done by a simple extension is half the product of the final stress and strain, and the total work is the sum of all such terms. In order that we may have the energy expression in the desired summation form, which will differ slightly from the engineering expression because of the factor $\frac{1}{2}$ in the engineering notation of strain, we shall derive the energy of deformation analytically.

Under the action of external forces an elastic body undergoes a deformation during which the forces do a certain amount of work stored in the body as *potential energy of deformation*. This deformation induces both stresses and strains in the body. It is naturally possible to express the deformation energy produced in terms of stress and strain and is then called *strain energy*.

In order to define this strain energy, it is necessary to consider the work expended in producing in an elemental domain, $dxdydz$, a strain,

$$\epsilon_{ij},$$

when subjected to the stress,

$$\sigma_{ij}.$$

The components of the force acting on the $y-z$ plane which is perpendicular to x, of the elemental domain are

$$\sigma_{xx}dydz,\quad \sigma_{xz}dydz,\quad \sigma_{xy}dydz,$$

and the final deformations corresponding to these forces are

$$\epsilon_{xx}dx,\quad \epsilon_{xz}dx,\quad \epsilon_{xy}dx.$$

An increment of work expended during the deformation by the forces across the $y-z$ plane is

$$dW_x=(\sigma_{xx}d\epsilon_{xx}+\sigma_{xy}d\epsilon_{xy}+\sigma_{xz}d\epsilon_{xz})dxdydz.$$

The increment of strain energy, which is the increment of work per unit volume is

$$dU_x=\sigma_{xx}d\epsilon_{xx}+\sigma_{xy}d\epsilon_{xy}+\sigma_{xz}d\epsilon_{xz}.$$

The increment of strain energy produced by the forces acting across the other principal planes have similar expressions, so that the net increment of strain energy dU induced by the strain is

$$\begin{aligned} dU&=dU_x+dU_y+dU_z\\ &=\sigma_{xx}d\epsilon_{xx}+\sigma_{xy}d\epsilon_{xy}+\sigma_{xz}d\epsilon_{xz}\\ &\qquad+\sigma_{yx}d\epsilon_{yx}+\sigma_{yy}d\epsilon_{yy}+\sigma_{yz}d\epsilon_{yz}\\ &\qquad\qquad+\sigma_{zx}d\epsilon_{zx}+\sigma_{zy}d\epsilon_{zy}+\sigma_{zz}d\epsilon_{zz}. \end{aligned} \tag{9}$$

In order to find the strain energy due to the total strain, it is necessary to integrate the above expression between 0 and the various e_{ik}'s. Substituting the σ_{ik} components by their values of Eq. (6) into Eq. (9) of the strain energy, we have

$$\begin{aligned} dU=&(2\mu+\lambda)[\epsilon_{xx}d\epsilon_{xx}+\epsilon_{yy}d\epsilon_{yy}+\epsilon_{zz}d\epsilon_{zz}]\\ &+\lambda[d(\epsilon_{yy}\epsilon_{zz})+d(\epsilon_{xx}\epsilon_{yy})+d(\epsilon_{xx}\epsilon_{zz})]\\ &+2\mu[\epsilon_{xy}d\epsilon_{xy}+\epsilon_{xz}d\epsilon_{xz}+\epsilon_{yz}d\epsilon_{yz}\\ &\qquad+\epsilon_{yx}d\epsilon_{yx}+\epsilon_{zx}d\epsilon_{zx}+\epsilon_{zy}d\epsilon_{zy}]. \end{aligned}$$

Integrating the above, and factoring out the stress-stain relationship, the strain energy reduces to

$$\begin{aligned} U=&\tfrac{1}{2}\sigma_{xx}\epsilon_{xx}+\tfrac{1}{2}\sigma_{yy}\epsilon_{yy}+\tfrac{1}{2}\sigma_{zz}\epsilon_{zz}+\tfrac{1}{2}\sigma_{xy}\epsilon_{xy}\\ &+\tfrac{1}{2}\sigma_{xz}\epsilon_{xz}+\tfrac{1}{2}\sigma_{yz}\epsilon_{yz}+\tfrac{1}{2}\sigma_{yx}\epsilon_{yx}+\tfrac{1}{2}\sigma_{zx}\epsilon_{zx}+\tfrac{1}{2}\sigma_{zx}\epsilon_{zx}. \end{aligned}$$

The energy per unit volume can be written concisely in summation notation as

$$U=\tfrac{1}{2}\sum_l\sum_m\sigma_{lm}\epsilon_{lm}. \tag{10}$$

ENERGY INDEPENDENCE OF DISTORTION AND DILATATION

In this section, we shall derive the actual stress and strain tensors of distortion and dilatation (the deviator and spherical tensors) on the basis of two postulatory conditions: (1) the

[6] S. Timoshenko, *Theory of Elasticity* (McGraw-Hill Book Company, Inc., New York, 1934), pp. 135–137.

stress and strain tensors can be decomposed into two and only two fundamentally different independent constituents, and (2) the total elastic strain energy is simply the sum of the strain energy of the individual constituents.

By fundamentally independent, we mean that the characteristics of the constituents, such as elastic moduli, yield point, and mechanical reversibility are dissimilar.

The first condition is reasonable to anticipate because of several experimental facts. They are:

(a) In "stiff" materials like glass or steel the resistance to both distortion and distension is high, yet in "supple" materials like rubber the resistance to dilatation is comparable to that of "stiff" materials while the resistance to distortion is negligible.

(b) The resistance to dilatation of a body is always greater than its resistance to distortion as evidenced, for instance, by the comparative magnitude of the bulk modulus and shear modulus of the theory of elasticity.

(c) Temperature has a marked effect on the resistance of a material to distortion. It affects the resistance to dilatation to a much lesser degree if at all. In fact, at the melting point the resistance to distortion practically vanishes while the resistance to distention does not.

(d) Recent experiments of Bridgman[7] show that the dilatation bodies subjected to hydrostatic pressure up to 20,000 atmospheres is elastic with no detectable permanent deformation. It is well known that the distortion of bodies produced by mechanical loading appears elastic up to a much lower limit than the above which limit decreases with the accuracy of the detecting instruments. Above this limit there is permanent deformation.

(3) The "conditions of plasticity" are properties of the stress deviator only.[8]

The second condition is really a different way of expressing the first, because the superimposed energy of two independent phenomenon is simply the sum of the energies of each phenomenon.

The first condition is expressed as

$$\sigma_{lm}=\sigma_{lm}'+\sigma_{lm}'' \tag{11}$$

$$\epsilon_{lm}=\epsilon_{lm}'+\epsilon_{lm}'' \tag{12}$$

and the second condition is expressed as

$$U=U'+U''. \tag{13}$$

Expanding the expression for the strain energy, Eq. (10), in terms of the stress and strain constituents, we have

$$\begin{aligned}U&=\tfrac{1}{2}\sum_l\sum_m(\sigma_{lm}'+\sigma_{lm}'')(\epsilon_{lm}'+\epsilon_{lm}'')\\&=\tfrac{1}{2}\sum_l\sum_m\sigma_{lm}'\epsilon_{lm}'+\tfrac{1}{2}\sum_l\sum_m\sigma_{lm}''\epsilon_{lm}''\\&\qquad+\tfrac{1}{2}\sum_l\sum_m(\sigma_{lm}'\epsilon_{lm}''+\sigma_{lm}''\epsilon_{lm}')\\&=U'+U''+U^x,\end{aligned}$$

where U^x is the cross product energy term containing both kinds of constituents. Substituting Eq. (6) for the stress, the U^x term for the non-principal orientation is

$$\begin{aligned}U^x&=\tfrac{1}{2}\sum_l\sum_m\{\mu\epsilon_{lm}'\epsilon_{lm}''+\lambda\epsilon_{lm}'(\textstyle\sum\epsilon_{kk}'')\delta_{lm}\\&\qquad+\mu\epsilon_{lm}''\epsilon_{lm}'+\lambda\epsilon_{lm}''(\textstyle\sum\epsilon_{kk}')\delta_{lm}\}\\&=\tfrac{1}{2}\sum_l\sum_m\{2\mu\epsilon_{lm}'\epsilon_{lm}''+\lambda[\epsilon_{lm}'(\textstyle\sum\epsilon_{kk}'')\\&\qquad+\epsilon_{lm}''(\textstyle\sum\epsilon_{kk}')]\delta_{lm}\}\\&=\mu\sum_k\epsilon_{kk}'\epsilon_{kk}''+\mu[\epsilon_{xy}'\epsilon_{xy}''+\epsilon_{xz}'\epsilon_{xz}''\\&\qquad+\epsilon_{yz}'\epsilon_{yz}'']+\lambda(\textstyle\sum\epsilon_{kk}')(\sum\epsilon_{kk}''),\end{aligned}$$

and for the principal orientation is

$$U^x=\mu\sum\epsilon_k'\epsilon_k''+\lambda(\textstyle\sum\epsilon_k')(\sum\epsilon_k').$$

By Eq. (13), U^x must vanish and the following equations must be satisfied:

$$(\epsilon_{xx}'+\epsilon_{yy}'+\epsilon_{zz}')(\epsilon_{xx}''+\epsilon_{yy}''+\epsilon_{zz}'')=0 \tag{14}$$

$$\epsilon_{xx}'\epsilon_{xx}''+\epsilon_{yy}'\epsilon_{yy}''+\epsilon_{zz}'\epsilon_{zz}''=0 \tag{15}$$

$$\epsilon_{xy}'\epsilon_{xy}''+\epsilon_{xz}'\epsilon_{xz}''+\epsilon_{yz}'\epsilon_{yz}''=0. \tag{16}$$

The first equation is satisfied for either

$$\textstyle\sum\epsilon_{jj}'=0,\quad\text{or}\quad\sum\epsilon_{jj}''=0.$$

Since the choice of the primed and double-primed constituents is arbitrary, we choose to write

[7] P. W. Bridgman, Proc. Am. Acad. Arts Sci. **74**, 21–51 (1940).

[8] A. Nadai, Reference 5, p. 105; R. v. Mises, Goethenger Nach., 1913; W. Prager, Reference 1, pp. 21–26.

Eq. (14) as

$$\epsilon_{xx}'+\epsilon_{yy}'+\epsilon_{zz}'=0. \tag{17}$$

Multiplying Eq. (17) by ϵ_{zz}'' and subtracting it from Eq. (15), ϵ_{xx}' is eliminated

$$(\epsilon_{yy}''-\epsilon_{xx}'')\epsilon_{yy}'+(\epsilon_{zz}''-\epsilon_{xx}'')\epsilon_{zz}'=0.$$

Since ϵ_{yy}' and ϵ_{zz}' are independent variables, Eq. (17) being the only relation between the ϵ_{jj}''s, it follows that

$$\epsilon_{yy}''-\epsilon_{xx}''\equiv 0, \quad \text{and} \quad \epsilon_{zz}''-\epsilon_{yy}''\equiv 0,$$

or

$$\epsilon_{ii}''\equiv\epsilon_{jj}'' \tag{18}$$

for all orientations. Now $\sum\epsilon_{jj}''$ is the first invariant, Eq. (1), of the ϵ_{ik}'' tensor. Hence $\epsilon_{ii}''=\epsilon_{jj}''$ is a constant under rotation. The second invariant, Eq. (2), of the ϵ_{ik}'' tensor, with the single subscripts referring to the eigenvalues or principal strains, is

$$\epsilon_x''\epsilon_y''+\epsilon_x''\epsilon_z''+\epsilon_y''\epsilon_z''=\epsilon_{xx}''\epsilon_{yy}'' +\epsilon_{xx}''\epsilon_{zz}''+\epsilon_{yy}''\epsilon_{zz}''-(\epsilon_{xy}''^2+\epsilon_{xz}''^2+\epsilon_{yz}''^2).$$

By Eq. (18)

$$\epsilon_x''=\epsilon_y''=\epsilon_z''=\epsilon_{xx}''=\epsilon_{yy}''=\epsilon_{zz}'',$$

and hence

$$\epsilon_{xy}''^2+\epsilon_{xz}''^2+\epsilon_{yz}''^2=0.$$

Since the strain components are all real, it follows that

$$\epsilon_{xy}''\equiv\epsilon_{xz}''\equiv\epsilon_{yz}''\equiv 0,$$

or

$$\epsilon_{ij}''\equiv 0 \ (i\neq j). \tag{19}$$

The third equation to be satisfied, Eq. (16), is therefore a necessary consequence of the first two.

In brief, the cross-product term U^x vanishes if and only if

$$\sum \epsilon_{jj}'\equiv 0 \tag{20}$$

and

$$\epsilon_{jj}''\equiv\epsilon_{ii}''. \tag{18}$$

A useful alternative way of writing Eq. (18) is

$$\epsilon_{ii}''=\tfrac{1}{3}\sum\epsilon_{kk}''. \tag{21}$$

Physically these conditions express the fact that the single-primed constituents involve no change in shape, and the double-primed constituents involve equal extensions in all directions.

The Lamé stress strain relationship may be used to find the expression of the stress constituents. Substituting Eq. (17) in Eq. (5), it follows that

$$\sigma_{lm}'=2\mu\epsilon_{lm}', \tag{22}$$

Summing Eq. (22) over all values of the normal stress components, we have

$$\sum\sigma_{ii}'=2\mu\sum\epsilon_{kk}=0. \tag{23}$$

The double-primed equivalent of Eq. (5) is

$$\sigma_{ii}''=2\mu\epsilon_{ii}''+\lambda\sum\epsilon_{kk}'' \tag{24}$$

and

$$\sigma_{jj}''=2\mu\epsilon_{jj}''+\lambda\sum\epsilon_{kk}''.$$

Subtracting these leaves

$$(\sigma_{ii}''-\sigma_{jj}'')=2\mu(\epsilon_{ii}''-\epsilon_{jj}'').$$

By Eq. (18)

$$\epsilon_{ii}''=\epsilon_{jj}'', \tag{18}$$

and therefore

$$\sigma_{ii}''=\sigma_{jj}'', \tag{25}$$

or

$$\sigma_{ii}''=\tfrac{1}{3}\sum\sigma_{kk}''. \tag{26}$$

For the same reason that Eq. (18) implies Eq. (19), Eq. (25) implies

$$\sigma_{ij}''\equiv 0 \ (i\neq j). \tag{27}$$

Summing Eq. (24) over all values of i produces

$$\sum\sigma_{ii}''=2\mu\sum\epsilon_{ii}''+3\lambda\sum\epsilon_{kk}''=(2\mu+3\lambda)\sum\epsilon_{kk}''.$$

Substituting Eqs. (22) and (26) into the above gives

$$\sigma_{ii}''=(2\mu+3\lambda)\epsilon_{ii}''. \tag{28}$$

Equations (22) and (28) show that the strain constituents are proportional to their corresponding strain constituents. These proportionality factors, 2μ and $(2\mu+3\lambda)$, are the elastic moduli of distortion and dilatation, respectively.

Equations (11) and (12) then become

$$\epsilon_{lm}=2\mu\sigma_{lm}'+(2\mu+3\lambda)\sigma_{lm}'',$$

$$\sigma_{lm}=\frac{1}{2\mu}\epsilon_{lm}'+\frac{1}{2\mu+3\lambda}\epsilon_{lm}''.$$

The Lamé equation can further be used to express the constituents of stress and strain in terms of the total stress and strain. Subtracting

Eq. (19) from Eq. (12), and Eq. (27) from Eq. (12) gives

$$\sigma_{ij}' = \sigma_{ij} \ (i \neq j), \tag{29}$$

$$\epsilon_{ij}' = \epsilon_{ij} \ (i \neq j). \tag{30}$$

The normal components from Eq. (11) and (12) are

$$\sigma_{ii} = \sigma_{ii}' + \sigma_{ii}'',$$

$$\epsilon_{ii} = \epsilon_{ii}' + \epsilon_{ii}''.$$

Summing the above equations over all values of i and subtracting from them Eqs. (20) and (23) leaves

$$\sum \sigma_{ii}'' = \sum \sigma_{ii},$$

or

$$\sigma_{ii}'' = \tfrac{1}{3} \sum \sigma_{kk} = \sigma_m, \tag{31}$$

and

$$\sum \epsilon_{ii}'' = \sum \epsilon_{ii},$$

or

$$\epsilon_{ii}'' = \tfrac{1}{3} \sum \epsilon_{kk} = \epsilon_m. \tag{32}$$

Recalling the definitions of the mean normal stress and strain, Eqs. (7) and (8), it follows from Eqs. (31) and (32) that the normal components of one of the stress and strain constituents are equal to ϵ_m and σ_m. Equations (11) and (12) can therefore be expressed as

$$\sigma_{ii}' = \sigma_{ii} - \tfrac{1}{3} \sum \sigma_{kk} = \sigma_{ii} - \sigma_m \tag{33}$$

$$\epsilon_{ii}' = \epsilon_{ii} - \tfrac{1}{3} \sum \epsilon_{kk} = \epsilon_{ii} - \epsilon_m. \tag{34}$$

Equations (25) and (31), also (19) and (32) can be written simultaneously using the Kronecker delta, giving the general expression of the second stress and strain constituents

$$\sigma_{ij}'' = \sigma_m \delta_{ij}; \quad \epsilon_{ij}'' = \epsilon_m \delta_{ij}.$$

Similarly, Eqs. (29) and (33), also Eqs. (30) and (34) can be written simultaneously giving the general expression of the first stress and strain constituents,

$$\sigma_{ij}' = \sigma_{ij} - \sigma_m \delta_{ij}; \quad \epsilon_{ij}' = \epsilon_{ij} - \epsilon_m \delta_{ij}.$$

The relations between the various components of the constituents can be summarized in matrix form as

$$\sigma'' = \begin{pmatrix} \sigma_m & 0 & 0 \\ 0 & \sigma_m & 0 \\ 0 & 0 & \sigma_m \end{pmatrix} = (2\mu + 3\lambda)\epsilon''$$

$$= (2\mu + 3\lambda) \begin{pmatrix} \epsilon_m & 0 & 0 \\ 0 & \epsilon_m & 0 \\ 0 & 0 & \epsilon_m \end{pmatrix}$$

and

$$\sigma' = \begin{pmatrix} \sigma_{xx} - \sigma_m, & \sigma_{yx}, & \sigma_{zx} \\ \sigma_{xy}, & \sigma_{yy} - \sigma_m, & \sigma_{zy} \\ \sigma_{xz} & \sigma_{yz} & \sigma_{zz} - \sigma_m \end{pmatrix} \neq 2\mu\epsilon'$$

$$= 2\mu \begin{pmatrix} \epsilon_{xx} - \epsilon_m, & \epsilon_{yx}, & \epsilon_{zx} \\ \epsilon_{xy}, & \epsilon_{yy} - \epsilon_m, & \epsilon_{zy} \\ \epsilon_{yz} & \epsilon_{yz} & \epsilon_{zz} - \epsilon_m \end{pmatrix}.$$

Conclusion: The decomposition of stress and strain into independent constituents, such that the elastic strain energy of these constituents are independent, is unique. These constituents represent two kinds of deformation, namely distortion and dilatation, and are called deviators and spherical tensors. The relation between corresponding stress and strain constituents is strict proportionality.

PROPORTIONALITY OF DILATATION AND DISTORTING TENSORS

In this section we shall derive the form of the actual stress and strain tensors of distortion and dilatation on the basis of two postulatory conditions: (1) The stress and strain can be decomposed into two and only two fundamentally different constituents, and (2) the corresponding stress and strain tensor constituents are related by a proportional constant.[10]

The first condition is given for the same reasons as in the preceding section.

The second condition is another way of expressing the first condition, provided one accepts cause-and-effect arguments. If a strain is proportional to a stress, i.e., if $\epsilon_{ik} = a\sigma_{ik}$, Hooke's law is valid for all components of the stress and strain. It is well known that this proportionality is not observed in the general strain-stress relationships as shown by Eq. (12) which indicates that each of the ϵ_{kk} terms is a linear function of all the σ_{kk} terms. The lack of a causal relationship is more forcibly demonstrated for the case of simple axial extension in which there is a lateral strain with no component of stress in the direction of that strain. The question arises, what causes the lateral strain? The concept of Poisson's ratio cannot explain away the above, for this number is experimentally determined and has no theoretical significance. The explanation pro-

[10] H. Brandenburger, Schweiser Archiv der Angewandten Wissenschieften und Tecknik 7, 223–235 (1941).

posed is that the observable stress and strain are not simple entities but are the superposition of two independent constituents. If these constituents truly are fundamental and distinguishable phenomenon, a causal relationship, i.e., a Hookean proportionality, exists between the corresponding components of the stress and strain constituents. The choice of two as the number of constituents follows from the fact that there are only two independent elastic parameters (see Eq. (11)).

The first condition is expressed as

$$\sigma_{lm}=\sigma_{lm}'+\sigma_{lm}'' \tag{11}$$

$$\epsilon_{lm}=\epsilon_{lm}'+\epsilon_{lm}'', \tag{12}$$

and the second condition as

$$\epsilon_{ik}'=a_1\sigma_{ik}' \tag{35}$$

$$\epsilon_{ik}''=a_2\sigma_{ik}'', \tag{36}$$

where a_1 and a_2 are different nonzero constants.

Expanding Eq. (35) we have

$$\epsilon_{xx}'=a_1\sigma_{xx}' \tag{37a}$$

$$\epsilon_{yy}'=a_1\sigma_{yy}' \tag{37b}$$

$$\epsilon_{zz}'=a_1\sigma_{zz}'. \tag{37c}$$

Expanding Eq. (36) we have

$$\epsilon_{xx}''=a_2\sigma_{xx}'' \tag{38a}$$

$$\epsilon_{yy}''=a_2\sigma_{yy}'' \tag{38b}$$

$$\epsilon_{zz}''=a_2\sigma_{zz}''. \tag{38c}$$

Adding Eqs. (37) we have

$$\sum \epsilon_{kk}'=a_1 \sum \sigma_{kk}'. \tag{39}$$

Subtracting pairs of Eqs. (38) we have,

$$\epsilon_{xx}''-\epsilon_{yy}''=a_2(\sigma_{xx}''-\sigma_{yy}'') \tag{40a}$$

$$\epsilon_{yy}''-\epsilon_{zz}''=a_2(\sigma_{yy}''-\sigma_{zz}'') \tag{40b}$$

$$\epsilon_{zz}''-\epsilon_{xx}''=a_2(\sigma_{zz}''-\sigma_{xx}''). \tag{40c}$$

Equation (39) is satisfied for any arbitrary values of the components and any value of the constant a_1 only when the sum of the components vanishes,

$$\sum \epsilon_{kk}''=0. \tag{41}$$

Equations (40) are satisfied for any value of the components, only when the components are equal,

$$\epsilon_{xx}''=\epsilon_{yy}''=\epsilon_{zz}''. \tag{42}$$

By using identical arguments of the preceding section, Eq. (41) coupled with the conditions of the first and second invariants, establishes that

$$\epsilon_{ij}''=0 \quad i\neq j,$$

and therefore

$$\epsilon_{ij}'=\epsilon_{ij}.$$

The values of the constants depends only on the stress-strain relationship, Eq. (6). Having shown that the form of the two strain constituents is the same as that found in the preceding section, similar arguments to those found between Eqs. (22) and (34) will establish that the forms of the constituents are identical to the stress and strain deviators and the stress and strain spherical tensors. It will be shown also that the proportionality constants are equivalent to those shown in Eqs. (22) and (28).

ACKNOWLEDGMENT

The author wishes to acknowledge the help of Professor Eugene F. Poncelet in the development of this paper. The ideas in the last section were in particular contributed by him.

1952 Summer Meeting of the Association

The 1952 Summer Meeting of the American Association of Physics Teachers will be held June 11–14 at the University of Iowa, Iowa City, Iowa, at the invitation of and in conjunction with the Colloquium of College Physicists. The four special June lectures, on interesting aspects of modern physics, will be given by Professor G. E. Uhlenbeck.

Contributed papers and demonstrations are welcome. Titles and abstracts should be sent to R. R. Palmer, *Beloit College, Beloit, Wisconsin.*

NOTES AND DISCUSSION

Can the Impact of a Falling Chain be Measured by a Balance?

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ABOUT forty years ago Horace Lamb in his *Dynamics* (my text is 1914, a sequel to an earlier publication) listed two problems which in modified form have been discussed by J. S. Miller[1] and John Satterly.[2] Lamb not only stated the problems; he gave the answers. But the answers he gave are not the answers to the problems recently discussed. The recent writers have assumed that they are, but Professor Satterly has shown experimentally that the theory is not correct.

Lamb's problem (No. 6, p. 149) is "a uniform chain hangs vertically with the lower end just clear of a *horizontal table*. If the upper end be released prove that *at any instant*$\cdots$the *pressure* on the *table* is three times the weight then on the table." Miller and Satterly substitute for *table* the pan of a balance and ask for the *maximum reading* of the balance. Lamb was careful to point out that his pressure (our force) was instantaneous and academic. But instantaneous pressures are not registered by ordinary balances. They might be by a piezoelectric gauge and oscilloscope.

(1) Consider the case of the whole chain above the pan (mass M, length L, and $m=M/L$), the bottom link very nearly in contact with it. Following the analysis set forth by Lamb and followed by Miller and Satterly, the momentum of an element dx reaching the pan is $(mdx)v$ and the force due to the impulse of striking the pan is $(mdx)v/\Delta t$, where Δt is the total duration of stopping that link. Is $dx/\Delta t$ equal to the v of arrival? Certainly it is not always. It could be changed completely by putting on the pan a thin sheet of rubber. But if we take the foregoing product to be equal to $mv^2=2mgL=2Mg$ for the last link then we conclude that the instantaneous impulse of the last link is equal to twice the weight of the chain. Suppose, now, that we have two chains of the same M, L, and m, but one chain with 100 links, the other with 1000. How does it happen that $(mdx)v$ for the last link of one chain is made, by this analysis, equal to that of the other? It is done by making $dx/\Delta t$ of one equal to that of the other, although the dx in one case is one-tenth of that of the other. So we might have the absurd result that the arrival of a link of mass one milligram produces the same pressure on the pan as the arrival of a mass of one gram falling the same distance.

(2) Let us turn to another method of solution. First, consider the simplest case—if the pointer of a spring balance is at 0 and a weight $W=Mg$ is above and very nearly in contact with the pan and is released, the pan will descend until the pointer is at x_2, where $x_2-x_1=x_1$ if $x_1=Mg/k$ is the reading for the steady state. So x_2, the deflection, is twice that due to the steady state. There is no impact so the energy relation holds. If x_2 is the maximum displacement, the kinetic energy there is zero and $Mgx_2=kx_2^2/2$ or $x_2=2Mg/k$. I had to work out this problem more than fifty years ago; thus—a beam of light suddenly falls on one of the light pressure vanes of a torsion balance. The maximum deflection, for no damping and no gas action, is twice that for the steady state. The differential equation proves this and also that the motion is simple harmonic about the new zero; moreover, the period is not altered.

(3) Now consider the vertical chain falling. There is constant impact until the whole chain is assembled on the pan. The principle of energy cannot be used until after this takes place but the principle of momentum applies. Suppose there is no rebound and that the pan does not move appreciably until the whole chain is assembled. The total momentum of the chain is $\int_0^L \sqrt{(2gx)}dx=M\sqrt{(2gL)}/3$. If all this momentum is taken up by a new mass $M+\Sigma M$ the new kinetic energy will be $4M^2gL/9(M+\Sigma M)$. Now this total momentum consists of early and later elements. To what extent do they add together? And the forces? Where are the forces of yesteryear or of a hundredth of a second ago? It will be shown that to the extent that an ordinary hook-and-bar tubular spring balance can be used to test this point, the momenta and the forces add together; the early and the later ones are in phase. But as tested by a two-pan, equal-arm balance, the phase difference must be taken into account.

Spring Balance Results.—I have not been able to use a spring balance for the case of the whole chain falling. But for the case in which half the chain hangs down from the hook and the end of the other half is drawn up level with the hook then released, analysis similar to the preceding shows that the total momentum is now $2M\sqrt{(\frac{1}{2}gL)}/3$. The total mass now in motion is $M+M_s$, where M_s is the mass of the hook, bar, and effective mass of the spring. For the 500 g spring balance used, 50 g produced a deflection of 1 cm. $M_s=43$ g and for the chain used $L=144$ cm, $M=86$ g.

If V_0 is the initial speed of the new mass, $3MV_0/2=\frac{2}{3}M\sqrt{(gL/2)}$ and the initial kinetic energy $=3MV_0^2/4=2MgL/27$. Hence, the deflection y is given by the relation $2MgL/27+Mgy=k\{(y+y_0)^2+y_0^2\}/2$. Hence, $y/y_1=0.5+\{4L/27y_1+0.25\}^{\frac{1}{2}}$. Here $y_1=1.72$ cm, the deflection due to a weight of 86 g, also $Mg=ky_1$. If we substitute numerical values we find $y=4y_1=344$ g. As y is measured from the zero when half the length of the chain is hanging down, we must add 43 g to give a total reading of 387.

Similarly the spring balance can be used for the case in which the chain is hooked at the midpoint and the two halves are brought up so that the two ends are on a level with the hook, then released. Also it can be used when a "last link," a single mass, a short cylinder of brass, is dropped a certain distance, in this case 20 cm. In the Lamb, Miller (L-M) theory this should give a deflection due to 2 Mg, where M is the mass of a cylinder of equivalent height of the same linear density. The theoretical and experimental results for a spring balance are here compared.

Chain hanging from spring balance half down, half up

L-M Theory	This theory	Experiment	Damping corrected
172	387	340	360
Same chain hooked at midpoint half down, half up			
172	303	267	283
Chain $M=202$ g, $L=80$ cm hooked at midpoint, half down, half up			
400	494	475	490
"Last link," 89 g dropped 20 cm			
1650	445	412	436.

Note that in the last item the momentum $=M\sqrt{(2gh)}$ and $\Sigma M \cong 3M/2$.

From the foregoing it is clear that if impact is to be measured by a spring balance the aforementioned theory is correct and the L-M theory is completely in error. However for another spring balance for which M_s is large the theoretical values would be smaller and so would be the experimental.

The Two-Pan Equal Arm Balance.—Here we come to an extremely crude device for registering a force of short duration. The mass of the moving system M_p, consisting of the masses of the pans, and the equivalent mass of the lever arms, is large compared to that of any mass which one would dare to allow to fall on one of the pans. Let us take the simplest case, a "last link," a single mass falling h cm on pan A covered with a thin sheet of rubber. The momentum is $M\sqrt{(2gh)}$. The new mass is M_1+M_p+M, where $M_1=2, 3$—$10M$ is the mass on pan B. If A is depressed and B rises y cm, then $(M_1-M)gy=M^2gh/(M_1+M_p+M)$. Here $M_p \cong 1500$ g. Let $M=90$ g, $h=20$ cm. Then, giving to M_1 various values, we can find y. A fine pointer was attached to pan B and its position could be read on a vertical scale even for a quick throw surely to an accuracy of 0.2 mm. But the jarring of the balance made this precision unnecessary. However, the experimental readings agreed rather closely for values of $M=200$ g ($y=8$ mm) to $M=1000$ g ($y=0.7$ mm).

For a chain falling on a pan the momentum is $\frac{2}{3}M\sqrt{(2gL)}$. The new kinetic energy if all the elements of momentum add together is $4M^2gL/9(M_1+M_p+M)$ and this equals $(M_1-M)gy$. Again we can give to M_1 various values and to M_p and M the values of 1500 and 86 g and compute y. The experimental values are considerably below these theoretical values especially as M_1 increases and we suspect that as M_1 increases a larger fraction of the momentum goes into the earth by way of the bearings. Also, we see another cause for the discrepancy.

The spring and two pan balances are quite different. In the former the small early momenta add together but in the latter they may be wiped out by the action of gravitation. It is evident that pan B will not rise from its support until its acceleration upward is greater than g. We might go through the process of finding the rate of arrival of the momentum, finding the length of L_0 of the chain necessary for that purpose, then computing the momentum by integrating from L_0 to L in place of 0 to L. Or we might suppose that between the impacts of the n and $n+1$ links the pan B would fall back δ cm to the rest position in which case we would have $(2(n+1)a/g)^{\frac{1}{2}}-(2na/g)^{\frac{1}{2}}=(2\delta/g)^{\frac{1}{2}}$, where $a=1.44$ cm. It seemed reasonable to take $n \cong 40$ and $L_1=57$ cm. But the experimental values are still too small; thus,

M_1(g)	Theoretical y(mm)	Experimental y(mm)
200	19	2
300	9	0.2
400	6	0.1

The conclusions are (1) that no satisfactory theory of the pan balance has been presented, (2) that the pan balance is an extremely crude device for this purpose, (3) that it does not indicate any precise value of the impact, certainly not $3M$.

[1] J. S. Miller, Am. J. Phys. **19**, 63 and 383 (1951).
[2] John Satterly, Am. J. Phys. **19**, 383 (1951).

Electronic Spark Timing Device

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In this journal Klaiber and Kington[1] have described an electronic timing device whose number of time marks per second is equal to the frequency of the ac mains supplying the apparatus. Robson[2] gives an example of how the spark timing device can be used for recording time marks in oscillographs.

A characteristic of both the foregoing time marking devices is that the number of spark time marks per unit of time is equal to the frequency of the power supply. The authors have built an electronic timing device whose number of time marks per unit of time is equal to or less than the frequency of the mains voltage. In the latter case it can be an arbitrary subharmonic of the mains' frequency.

The timing device to be described operates on the following principles: The high frequency and high voltage currents required to produce the spark are supplied by a Tesla transformer. There is a condenser connected in series with the primary coil of the Tesla transformer, both lying in parallel with a thyratron tube. Owing to the constant negative bias of the thyratron grid this circuit is open. If the condenser is charged and then discharged by allowing current to flow through the thyratron, the Tesla transformer will produce a high voltage spark. The grid of the thyratron tube is controlled by means of a multivibrator. The multivibrator produces positive voltage pulses larger than the negative bias of the thyratron tube grid, in consequence of which the tube is made to fire. The frequency of the multivibrator can be varied within wide limits, being equal to any of the subharmonics of the ac frequency. A spark, of course, will occur only in that half-period in which the multivibrator produces a positive voltage pulse.

The wiring diagram of the apparatus is shown in Fig. 1. The timing device consists of three main parts indicated by numbers in circles in Fig. 1. The parts are

(1) Thyratron controlled firing circuit
(2) Pulse generator
(3), (4), (5) Voltage supplies.

Voltage supply (3) charges the condenser C_1 in the anode circuit of the thyratron tube to about 2400 v. From voltage supply (4) a negative potential divided on the resistors R_4 and R_5 biases the thyratron tube to cutoff. If now the grid of the thyratron tube obtains a positive voltage pulse

sufficiently large for the tube to fire, the condenser in the anode circuit of the thyratron will discharge through the tube and the primary coil of the Tesla transformer T. The current flowing through the primary coil of the Tesla transformer T induces high frequency and high voltage currents in the secondary coil. This results in discharging a spark in spark gap S.

In the circuit shown in Fig. 1 the pulse generator marked (2) is a modification designed for our purpose of the basic

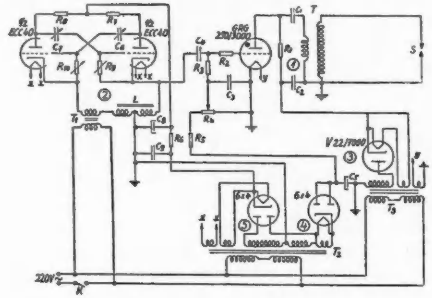

FIG. 1. Electronic spark timing device. Wiring diagram.

C_1 =0.05 microfarad 3500 v
C_2 =0.5 microfarad 3500 v
C_3 =1.0 microfarad 500 v
C_4 =0.05 microfarad 1500 v
C_5 =16 microfarad 450/500 v (electrolytic)
C_6 =variable according to Table I
C_7 =variable according to Table I
C_8 =16 microfarad 450/500 v (electrolytic)
C_9 =4 microfarad 500 v
L =iron-cored twin choke 20+20 h min, 500+500 ohm max
R_1 =0.03 megohm, 12 w, wire resistor
R_2 =0.01 megohm, 1 w, carbon resistor
R_3 =0.05 megohm, 1 w, carbon resistor
R_4 =0.02 megohm, 2 w, potentiometer
R_5 =0.05 megohm, 3 w, carbon resistor
R_6 =5000 ohm, 6 w, wire resistor
R_7 =0.2 megohm, 1 w, carbon resistor
R_8 =0.2 megohm, 1 w, carbon resistor
R_9 =variable according to Table I (carbon resistor)
R_{10} =variable according to Table I (carbon resistor)
ECC40 =twin triode
GRG 250/3000 =thyratron tube
V22/7000 =rectifier tube
6x4 =rectifier tube
T =Tesla transformer. Primary: 10 turns on 110-mm diameter base, wire diameter 3 mm. Secondary: 250 turns on 80 mm diameter base, wire diameter 1 mm.
T_1 =transformer 220/12 v, primary inductance 25 h min
T_2 =transformer 220/2×320 v, 40 ma max, 3×6.3 v 1 amp max
T_3 =transformer 220/1600 v, 20 ma max, 6.3 v, 1 amp max, 2.5 v, 5 amp max
S =spark gap.

TABLE I. Values C_6, C_7, R_9 and R_{10} belonging to the different marking frequencies of the electronic spark timing device.

Interval between two marks	C_6 (μf)	C_7 (μf)	R_9 (meg)	R_{10} (meg)
20 msec			0.2	0.2
40 msec	0.02	0.017	0.6	0.4
100 msec			1.8	1.6
200 msec			0.6	0.4
400 msec	0.1	0.085	1.0	1.2
1 sec			4.0	4.0
2 sec			0.6	0.4
4 sec	1	0.85	1.0	1.2
10 sec			4.0	4.0

circuit of the well-known multivibrator. The circuit employed here differs from the conventional type in containing the center-grounded reactance L in its cathode circuit. This modification is required because in synchronizing the multivibrator of conventional circuit the negative voltage pulses in its output are large in amplitude while the amplitude of the positive voltage pulses required to fire the thyratron is small. By the change made here large positive voltage pulses can be obtained if the output connects to the cathode of the multivibrator tube. The multivibrator is synchronized from the 50 cps mains through transformer T_1. The synchronizing voltage is fed into the left-hand tube cathode circuit of the multivibrator. When synchronized, this circuit can produce voltage pulses only at integral subharmonics of the mains' frequency. In multivibrator circuit (2) the values of the elements C_6, C_7, R_9, and R_{10} are chosen so that the multivibrator operates at a desired frequency. Appropriate values of these components are listed in Table I.

A differentiating network C_4-R_3 connected to the cathode of the right-side tube of the multivibrator transforms the voltage pulse obtained from the tube into a very steep positive voltage pulse. The GRG-type thyratron tube is controlled by this pulse being applied to its grid.

The direct voltage required to operate the multivibrator is provided by voltage supply (5).

A more detailed description of the apparatus will appear later.

[1] G. S. Klaiber and L. K. Kington, Am. J. Phys. 18, 397 (1950).
[2] R. C. Robson, J. Sci. Instr. 28, 60 (1951).

AMERICAN JOURNAL OF PHYSICS VOLUME 20, NUMBER 4 APRIL, 1952

LETTERS TO THE EDITOR

Cooperation of High Schools and Colleges on Problems of Physics Teaching

AT the Chicago meeting of the American Association of Physics Teachers, I was asked to make a few remarks concerning the activities of the Southern California Regional Section of the AAPT which relate to the subject of cooperation of the secondary school and college teachers of physics on problems of physics teaching.

The importance of this subject was recognized by the Section from the very beginning of its formation in January, 1945. The need for cooperation between teachers in the high schools and colleges was brought clearly before the Section through the efforts of Mr. Roy McHenry, a long-time member of AAPT, a charter member of the Section, and at the time of the Section's formation a teacher in the Santa Monica Public High School with many years of successful teaching experience at the secondary

school level. Mr. McHenry served as Secretary of the Section for several years and as President for one year. He has been the keystone of the Section's activities relating to the mutual interests of high schools and colleges. I take this opportunity to pay tribute to Mr. McHenry for all that he has done.

I think you will be interested in hearing something about the structure and activities of the Section as they relate to the question under discussion. Let me list a few facts.

The Executive Committee of the Section is composed of the President, a Vice-president for Junior Colleges, a Vice-president for High Schools, a Vice-president for Colleges, the Secretary-Treasurer, and the president just retired. This elected group is responsible for planning the semi-annual meetings held in the fall and in the spring and for making the scholarship awards to the winners in the Annual Competitive Physics Test for High School Students. I want to emphasize the importance of having each group represented on the Executive Committee. The office of Vice-president for High Schools can only be held by a high school teacher. This has done much to bring the high school teacher group into Section participation by insuring that meetings include items of interest to high school teachers. It also gives tangible proof of the importance of the high school group to the Section. We have found that high school teachers are a little shy about contributing papers at our meetings. Fortunately this lack of confidence is rapidly disappearing. A definite effort has been made to seek out papers from this group and an examination of our past programs will show that we have averaged better than one paper by a high school teacher at each of our meetings. As a matter of fact, at our meeting last March, two high school students presented an excellent paper. Incidentally, the papers presented by the high school teachers have more often than not been the most interesting on the program. On one occasion, several years ago, we devoted an entire morning session to a panel discussion on the subject which we are considering this morning and on the problems of high school physics teachers. It was most worthwhile. From this particular discussion it became evident that there was a need for a study of the teaching loads and schedules of high school and junior college teachers. This study was sponsored by the Section and a committee spent much time and effort on the task. I doubt if the results of this study will be published but no doubt a great deal of good was accomplished by bringing the existing conditions into the open.

The Eighth Annual Competitive Physics Test for High School Students will be held next May. This test is sponsored by the Section and is administered by the Test Committee. The examination is two hours in length and consists of selected problems from the area of general physics covered in high school courses. The problems are graded on the basis of method as well as on correct answers. Any high school student in Southern California may compete. There is no team participation. Several years ago team competition similar to that in the test given by the American Chemical Society was tried but abandoned as undesirable. The object of the test is to encourage a wide interest in physics and wide participation. It is also intended as a yardstick for gaining an understanding of over-all teaching effectiveness. The test is given at one of the colleges or universities in the greater Los Angeles area and also at Redlands, San Diego, and Santa Barbara. The three last-mentioned locations are more or less on the periphery of the Southern California area and reduce the transportation problem for students at a considerable distance from Los Angeles. Early in January announcements of the test are mailed to all private and public high schools, about two hundred and fifty in number, in the Southern California area. Approximately two hundred students from more than fifty high schools compete. The tests are graded by the Test Committee which reports the scores to the Executive Committee. A list of the names of all participating students with each score and the name of their school is mailed to each high school. Specially printed certificates of award are sent to each of the fifteen top ranking students. Duplicate certificates are also sent to their schools for display. In addition to the certificates scholarships are awarded by the following schools: California Institute of Technology, Occidental College, Pomona College, Redlands University, University of California, University of Southern California, and Whittier College. These scholarships are worth all or the most part of a year's tuition and are assigned by the Executive Committee on the basis of rank and student choice of school. It has become a custom for the high school teachers in the Los Angeles area to accompany their students to the test. A special program is arranged for the teachers. As part of this program copies of the test with correct solutions of the problems are presented to the assembled teachers for comment and discussion. The teachers have been most enthusiastic about these meetings.

Both the Los Angeles County Schools and the Los Angeles City Schools have a Teacher's Institute. These institutes consist in part of a large list of lectures, meetings, forums, and workshops approved by the school administrations for teacher attendance on a quasi-compulsory basis. Teachers are expected to attend a certain number of these meetings during the year and receive credits for doing so. Our fall meetings which are always held at one of the colleges or universities in the Los Angeles City area have for many years been listed in the printed Institute Bulletins and carry credit. This has encouraged many high school teachers to attend our meetings and we have had as many as a hundred at some of our meetings. This is most valuable in that they see and hear about the activities of the Section. Such occasions afford the presiding officer the opportunity of pointing out the interest of the Section in the problems of high school teaching.

These are the things which we are doing in the Southern California Regional Section which relate to the topic under discussion. There is much more that needs to be done. A check of our current membership list shows that we have 107 members in the Section. Of these, 37 are high school teachers, and of these only 5 are members of the

AAPT. In my opinion the basic requisite for cooperation between high school teachers and college teachers in any region is that there shall be high school teachers actively participating in the AAPT regional sections and enjoying full membership in the AAPT. Teachers of physics at the secondary school level must be permitted and encouraged to assume their responsibilities in the task of "the advancement of the teaching of physics and the furtherance of appreciation of the role of physics in our culture." We must remember that with few exceptions it is the high school teacher who first brings the growing student into contact with physics. What he does and how he does it is of great concern to all of us who teach at the college level. We must understand his problems and contribute all we can to synthesize the entire physics teaching process in order to further the object of the AAPT.

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Chromatic Aberration in the Eye

A STRIKING demonstration of chromatic aberration in the human eye, one which a whole class of students may readily perform for themselves in their seats, requires no apparatus other than the observer's finger, his eye, and an ordinary light source.[1]

The phenomenon may be observed by closing one eye and focusing the other on the boundary of a light source. A uniform source, such as a glass bowl over a classroom light, works well, although daylight viewed at a window

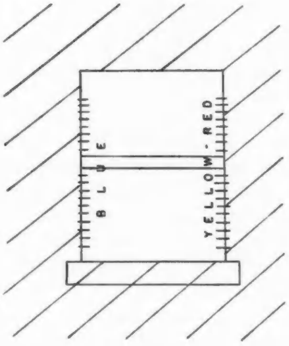

FIG. 1. One eye closed, open eye focused on window; finger near eye introduced from right side produces colored boundaries indicated.

frame will suffice. At the same time place a finger (or opaque obstruction) in front of the open eye in such a way that its out-of-focus image just begins to introduce an apparent distortion of the image of the boundary being focused upon. If the obstruction is introduced from the right, the red end of the visible spectrum appears outlining the right-hand boundary of the light source. As the finger moves across the eye, the blue end of the visible spectrum

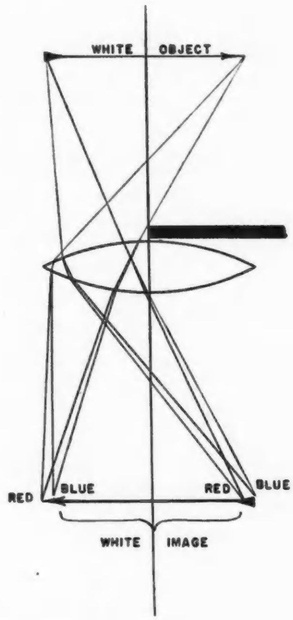

FIG. 2. Chromatic effect reproduced with uncorrected lens.

appears outlining the left-hand boundary of the light source (Fig. 1). It is difficult to see the blue using an incandescent source since such a source emits little energy in the blue region of the spectrum, and the eye is relatively insensitive in this region.

An explanation of the source of these colors can be made with the help of an additional experiment. An ordinary convergent lens used with some sort of screen serves to simulate the action of the eye. Repeating the foregoing procedure, again the colors are apparent, but this time it is a simple matter to substitute a comparable achromatic lens. This substitution eliminates the colors.

The way chromatic aberration accounts for the observed colors may be made clear to the class by considering the lens as a double prism. As shown in Fig. 2, the blocked-off lens refracts the blue and red differently, as would a prism. The colors supplement one another in the central part of the image, producing white. At the edges, however, such addition does not occur and the colors are apparent. The second half of the lens, if uncovered, would produce the same aberration, but in the reverse direction. Thus, with the unobstructed lens the end effects add to make the entire image white.

Incidentally, these two experiments can be used to illustrate the fact that the image on the retina is reversed. The class's subjective observations will agree with our simulated eye results except that one is reversed.

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[1] For a related experiment see D. H. Jacobs, *Fundamentals of Optical Engineering* (McGraw-Hill Book Company, Inc., New York, 1943), p. 84.

Note on the Teaching of Geometrical Optics

WE have observed that almost all the books that deal with geometrical optics use the designation of a segment $AB=x$ by the letter x in a line with two arrows, one at each end. One of the difficulties of the study of the subject is exactly the diversity of convention of signs among the excellent treatises available, and we have adopted the use of designating the segment $AB=x$ by a line with ***only one*** arrow, at its extremity. This has the advantage of emphasizing not only the origin, but also the sign of the segment, according with the propagation of the rays of light.

The use of this notation in the figures, and the adoption of the convention of signs and notation on geometrical optics now recommended by the International Commission of Optics,[1] have reduced to a minimum the doubt and confusion in the lectures and exercises.

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[1] Am. J. Phys. 19, 122 (1951).

AMERICAN JOURNAL OF PHYSICS VOLUME 20, NUMBER 4 APRIL, 1952

ANNOUNCEMENTS AND NEWS

Book Reviews

Heat and Thermodynamics. Third Edition. MARK W. ZEMANSKY. Pp. 465+xiv, Figs. 183, 16×23.5 cm. McGraw-Hill Book Company, New York, 1951. Price $6.00.

The new third edition of Zemansky's excellent book is about 70 pages longer than the second edition. The additions include: Giauque's temperature scale; sections on dielectrics and piezoelectricity; Grüneisen's equation (while a section on Richardson's thermionic equation has been omitted); a Chapter XVI on the Physics of Very Low Temperatures containing an expanded study of the liquefaction of gases and a section on the third law. The material on transfer of heat has been redistributed; sections on Poiseuille's equation and Reynolds number have been added. The mathematically inclined reader will be happy to find in Chapter X (Entropy) a new brief section on the Principle of Carathéodory which postulates that in the neighborhood of any state of a system there exist adiabatically inaccessible states. Via integrability conditions, Carathéodory's principle yields the existence of an entropy function and of the absolute temperature. A sketch of the proof of this fact is included.

If in glancing through the new edition of this much-used book the reviewer feels one regret, it is that the author has not used his great authority to break with some anachronistic traditions of symbolism. In its differential form, the first law is, as almost everywhere, stated as follows:

$$dQ=dU+dW.$$

Before formulating it the author declares that an infinitesimal amount of heat is an inexact differential to be represented by the symbol dQ. But there is no discussion of the general notion of a differential. If it is admitted that a textbook should present to the reader the easiest access to the subject matter, it is hard to see why texts on thermodynamics should not include, at least as an alternative, the formulation

$$\dot{Q}(t)=\dot{U}(t)+\dot{W}(t) \text{ at any moment } t,$$

where the dot is the symbol for an instantaneous rate of change. This latter formulation of the first law is applicable only to "differentiable" processes (in which Q, U, and W *admit* instantaneous rates of change). But it has the advantage of introducing only mathematical concepts with which even every beginner is thoroughly familiar. It cannot be said that these latter concepts include the general notion of a differential or the somewhat obscure idea of inexact differentials. In a beginner one cannot presuppose a thorough understanding of more than the idea of the differential of a given function.

The time seems also ripe for the explicit introduction of the Stieltjes integral into textbooks of thermodynamics. This concept can be defined in a few words. Its importance in mechanics is increasing, and it has become a standard tool of statisticians. The Stieltjes integral is clearly designed to supersede the eighteenth century idea of inexact differentials.

KARL MENGER
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Thermodynamics. Fourth Edition. A. W. PORTER. Pp. 124+vii, Figs. 21, 11×17 cm. John Wiley and Sons, Inc., New York, 1951. Methuen and Company, Ltd., London. Price $1.50.

Porter's little book, one of Methuen's Monographs on Physical Subjects, is especially valuable because of the perspective which it gives the student on the development of the fundamental ideas of thermodynamics. This is of special importance in this field in which the ideas are so abstract that the student may only gradually come to understand their full implications.

The new fourth edition is characterized by the addition of two new appendices by Professor W. Wilson. The first of these is designed to replace a revision of the text itself. Wilson writes "Porter's little book is in some ways unique, and it seemed to me, therefore, undesirable to interfere with the text, except to correct the few obvious misprints in it." This procedure has both advantages and disadvantages from the standpoint of the student. Wilson's comments fill somewhat the same function as the commentary of a class teacher, illuminating obscure points from a different point of view. The student will find the commentary on these points more useful than another textbook; the comments are generally to the point, in the language of the original text, and genuinely complementary.

The disadvantage of the separate appendix is that the text and appendix must literally be read together as Wilson intends to gain the full advantage of the commentary. This is especially true in sections where the original text may actually be misleading. Since no references to the appropriate sections of the appendix have been placed in the text this is at best inconvenient. Additional cross-references within the body of the text itself would also be useful especially for formulas reintroduced in later sections for specific applications or otherwise.

The opening chapters with their development of the fundamental principles seem to this reviewer the most valuable feature of the book. However the sections on applications of thermodynamics are necessary to give students an appreciation of the value and power of the subject, and some weaknesses in the original text have been clarified in Wilson's appendix. His second appendix adds a brief discussion of the application of thermodynamics to black body radiation.

Porter's "Thermodynamics" should be a very useful book for the student beginning the study of thermodynamics who wishes more insight into the nature of its fundamental concepts and their historical development that is given by many of our intermediate thermodynamics texts.

ROBERT H. FROST
University of Missouri

Tensor Analysis: Theory and Applications. I. S. SOKOLNIKOFF. Pp. 335+ix, Figs. 43, 6×9¼ in. John Wiley and Sons, Inc., New York, 1951. Price $6.00.

This book was obviously written by a skilled teacher. It is well known that knowledge and enthusiasm are necessary but not sufficient for good teaching; a third requirement, at least, must also be satisfied. The teacher must be able to anticipate some of the questions which arise in the student's mind as the exposition progresses. The good teacher arranges his presentation to answer these questions as they arise without impeding the flow of the argument.

Professor Sokolnikoff, in this book, illustrates admirably the good teacher in action. Perhaps his twenty-five years of experience as a teacher contribute to this end, but such experience has obviously not led to the same result in all other authors. For example, any tensor analysis text will tell you that a tensor of weight one is a tensor density. Professor Sokolnikoff (p. 72) tells you why. Again (p.155) the author is seeking solutions of a particular class; a footnote explains why that class is desirable.

About 1865 an English mathematician inquired plaintively "What is the use of a book without pictures?" Professor Dodgson could not condemn this book on such grounds—it has 43 figures. The geometrically minded reader will wish there were more.

The book spends the first chapter (50 pages) on linear vector spaces and matrices, the second chapter (55 pages) on tensors. This completes the formal development of the mathematics. Students will appreciate this isolation which permits them to look up an item of mathematics without becoming lost in the applications. The chapter on linear vector spaces and matrices clearly has in mind that the reader may apply them to quantum mechanics. The presentation includes more items of use to quantum mechanics than the usual text on vector analysis. On the other hand, the vector operators are not developed in Chapter One, although they appear later among the applications. The reader who needs information on the more elementary aspects of vector analysis will require another source.

The last four chapters treat applications to geometry, mechanics, special and general relativity, fluids and elasticity. Fluids are treated in full nonlinear form with reduction to the special case of linearity. A brief historical background is given for the major problems treated. There is an adequate supply of problems, many of the form "Prove the following. . . ." This will be appreciated, especially by those who must work at the subject without an instructor. In fact, the book as a whole is one of the few presentations which could be enthusiatically recommended to the solitary learner.

The development makes frequent references to original papers and to more extensive treatments of special topics. This procedure has a double utility. It permits a rapid and concise presentation of the main ideas while at the same time showing the relationship to particular areas of effort and to the historical course of evolution. One item of historical trivia is missing from the book. Perhaps the reviewer is neurotically sensitive to the semantic overtones of words, but what, if anything, about a tensor is tense? Probably the word was borrowed from Hamilton who used the word "tensor" for what we would now call the "length" of a quaternion. A quaternion (and its conjugate) made up a two-step operator; a "versor" which rotated a vector, and a "tensor" which extended it. This usage probably

antedates any idea of the tensor as a measure of stress, but an authoritative settlement of the question would be desirable.

It is, fortunately, seldom necessary for a reviewer to mention the printing, paper, and binding of a modern book from a major publishing house. Their excellence may be taken for granted. A book on tensor analysis with its jungle of subscripts and superscripts makes a fine hunting ground for the sort of person who delights in bagging a collection of misprints. There are about 10^6 symbols in this book. Despite every effort to the contrary, some of them are probably wrong. It is suggested that a good deal of pleasure and profit can be accrued by perusal of the symbols which are right.

M. J. WALKER
University of Connecticut

Astrophysics: A Topical Symposium. Edited by J. A. HYNEK. Pp. 703+xii, Figs. 136. McGraw-Hill Book Company, Inc., New York, 1951. Price $12.00.

This volume is an outgrowth of the celebration of the fiftieth anniversary of the Yerkes Observatory. In 14 chapters it reviews a large part of the field of astrophysics today. Each chapter is a self-contained, authoritative review of a subdivision of the field, at an advanced level, by an expert in the specific area treated.

General principles of classification of stellar spectra are first discussed by Kennan and Morgan. The second chapter, by L. H. Aller, is a concise summary of the physical theory of intensities and profiles of stellar absorption lines, and concludes with a section on cosmic abundance of the elements. Struve's chapter on peculiar stellar spectra contains a wealth of descriptive material on bright-line stars, close binaries, stars with extended atmospheres, novae, and some other variable stars. P. Swings gives a compact summary on molecular spectra in astronomical sources. The first part of the volume is completed with a chapter by B. Stromgren on physics of the stars. Although a number of topics are the same as in Aller's chapter, Stromgren's approach is essentially historical, and the two chapters supplement one another with little real duplication.

The second part contains three chapters, by E. Pettit, N. T. Bobrovnikoff, and G. P. Kuiper, respectively. Pettit's is chiefly descriptive of solar phenomena, and includes the measurement of solar and stellar radiation. Bobrovnikoff discusses the physical nature of comets. This is a very welcome summary, for it is based in part on material that has long been scattered in the literature—some of it in obscure places. Kuiper discusses the origin of the solar system, presenting the first printed account of his modification of Weizsacker's hypothesis. This is given in great detail and with less descriptive exposition than one would expect in a book of this sort.

In the third part, three of four chapters deal with binary stars: visual doubles, by G. Van Biesbroeck; spectroscopic binaries, by Hynek; and eclipsing stars, by N. L. Pierce. Van Biesbroeck includes a summary on stellar parallaxes, and Hynek discusses composite spectra and some statistics of binaries. The chapter on intrinsic variable stars, by Cecilia Payne-Gaposchkin, represents a new approach based on the recently recognized "Population Types I and II." Her diagrams of phase relations of phenomena in Cepheids merit careful study.

Part IV is especially outstanding. Greenstein's chapter reviews both observational and theoretical aspects of interstellar matter in the most complete single account that can be found in print. Chandrasekhar's chapter on stellar structure shows this author at his best. A forbiddingly mathematical subject is presented in a remarkably lucid manner.

The book as a whole is well-balanced, both in the distribution of the separate topics and in the relative emphasis of observation and theory. If there is a deficiency, it is perhaps in the assignment of less than a full chapter to the sun, a subject that might well occupy two or three chapters. The book was not designed as a text and it would require extensive supplements if it were used as one. But as a reference volume it is outstanding. Its usefulness is enhanced by extensive bibliographies at the end of nearly every chapter. The volume should be required reading for all graduate students in astronomy and astrophysics, and all workers in the field will find a great deal of value in its pages.

DEAN B. MCLAUGHLIN
University of Michigan

Initiation and Growth of Explosion in Liquids and Solids. F. P. BOWDEN AND A. D. YOFFE. Pp. 102, Figs. 73. Cambridge University Press, New York, 1951. Price $3.50.

To the experimental or theoretical research physicist anxious to gain a toe hold on the fascinating field of explosions this reviewer heartily recommends this small monograph. The general student of physics too can gain a real insight into the methods of proof by experiment so ably and interestingly presented. My chief regret is that the important information contained in this volume was not known much earlier. If it had been, the outright superstition and mysticism extant among war workers with explosives might have been dispelled to some extent and safety practices might have been reasonably improved.

The authors' main thesis in the book is to show that initiation of explosion, other than by direct thermal ignition, almost invariably takes place by local heating. This is accomplished through the intermediary of various mechanisms, chief among which are ordinary friction and adiabatic heating by compression of small entrapped gas pockets. Several series of ingenious experiments are described to prove this point, and further show that the "momentary hot-spot" temperatures required are appreciably higher than the usual steady thermal ignition temperatures. This is interpreted as evidence for the theory of Rideal and Robertson (Proc. Roy. Soc. (London) **A195**, 135 (1948)) that explosion spreads from the hot-spot by self-heating, namely, when the heat evolved during de-

composition is greater than the heat lost by conduction. The authors point out furthermore that the theory advanced by earlier workers must be supplemented. For not only is the hardness of grit particles present in solid explosives instrumental in initiating explosions, but a factor of prime importance is the melting point of the grit particles. In fact, by introducing grit particles of known melting point it was possible to estimate the temperature rise needed for initiation. There are some indications that the thermal conductivity of the grit particles is also a factor. The hardness of the particles is seemingly effective since (friction) stresses are concentrated at one or two points where the temperature rises to the necessary magnitude.

In the final chapter there are described electronic and photographic observations concerning the way in which the explosion develops into a large-scale detonation. It is shown that "with all the secondary explosives the reaction begins as a comparatively slow burning which accelerates, and after traveling a short distance passes over into a low velocity detonation. Some primary explosives such as mercury fulminate and lead styphnate behave in the same way." It is certainly most interesting to observe how the authors prove most of their points by performing experiments with the most elementary of apparatus and utilize more complicated techniques only when absolutely necessary. This chewing gum and baling wire point of view is extremely refreshing in the age of cyclotrons.

GEORGE E. HUDSON
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Quantum Theory of Matter. JOHN C. SLATER. Pp. 528+xiv, Figs. 111, 16×23.5 cm. McGraw-Hill Book Company, Inc., New York, 1951. Price $7.50.

In 1900 Max Planck introduced the hypothesis of quanta, which marked the beginning of quantum theory. This theory was very fruitful and finally developed into quantum mechanics and quantum electrodynamics. Whereas quantum theory before 1925 was deficient and unsatisfactory in various ways, quantum mechanics is logically complete and is a finished part of physical theory in the same sense that Newton's mechanics and classical electromagnetism are finished parts of physical theory. The development of quantum mechanics has made it possible for the first time to give a satisfactory theoretical explanation of the structure of molecules, chemical binding, and the theory of the solid state as well as some aspects of the liquid state. This explanation is in principle quantitative as well as qualitative, but the detailed mathematical work can be carried out completely or to a high degree of approximation only in the simpler cases. In other cases we must be satisfied with less accurate approximations and explanations using physical and mathematical thinking, generalizing what we learned in the simpler cases.

This book covers the field of quantum mechanics and its application to atoms, molecules, and the solid state. The basic plan of the book is logical and deductive, although the author gives a certain amount of the historical development of the subject in order to give the reader a sense of history and an appreciation of the fact that this subject is still developing. This reviewer believes that an understanding of how the subject evolved is very important. Although the treatment is mathematical, the amount of mathematical manipulation is kept to a minimum and the emphasis is on physical and mathematical thinking. Some of the mathematics is put into appendices of which there are twenty-two. The book is a very pleasant book to read because of its clarity.

In the first 200 pages the author presents quantum mechanics as well as the quantum theory of Bohr, and their application to some of the usual problems of the atom, rotator, linear oscillator, and potential wells. Perturbation theory is treated very thoroughly because it is so useful in the rest of the book. The treatment is elegant. The author presents general ideas in simple and concrete form. A method often used is to show how the general principle works out in a very simple case or cases, always keeping the mathematics as simple as possible. Thus, in discussing the analogy between geometrical optics and classical mechanics, pointed out by Hamilton, he considers some special cases with the aid of diagrams and simple mathematics. He finally ends up with Hamilton's equation for a particle, but these are the most highbrow equations in the whole discussion.

Although the student learns about orthogonal functions, solving wave equations, and so forth, the mathematical developments are kept as simple as possible with the aid of considerable verbal explanation. There is no reference to Hilbert space and other highbrow mathematics that can be dispensed with and still retain clarity. The WKB method is presented in a very clear manner.

One of the important chapters deals with the central field model for atoms, in which the author presents the Hartree self-consistent field method. This is important because the self-consistent field method forms the basis for a great deal of the subsequent discussion of molecules and solids.

The author uses the hydrogen molecule and molecule-ion to illustrate and explain the two methods of approaching the subject of molecular structure and chemical binding, namely, molecular orbitals and the Heitler-London method. The treatment is excellent, but he does not refer to the most accurate work on H_2 because he is interested in presenting methods and concepts that can be used in more complicated cases. The physics is kept in the foreground.

The treatment of chemical binding and valence is very clear. It is, of course, mathematical only in the sense that the author speaks about molecular orbitals, s and p atomic wave functions, perturbation calculations, applying the Heitler-London method, and using diagrams and graphs when necessary.

Other subjects treated in the book are the metallic state; mechanical, thermal, and chemical properties of matter;

electrical conductivity including semiconductors, dielectrics; and magnetism including superconductivity.

The book is very carefully written and free from errors. However on page 77 the use of v for velocity and dv for element of volume in the same discussion may cause some temporary confusion.

This book is a textbook with problems at the end of each chapter. Although it was written in part with the needs of chemists and metallurgists in mind, it will be very useful to physicists also.

Enos E. Witmer
University of Pennsylvania

New Members of the Association

The following persons have been made members or junior members (*J*) of the American Association of Physics Teachers since the publication of the preceding list [Am. J. Phys. 20, 191 (1952)].

Ames, Clarence Emerson, 2442 Birdsall St., Blue Island, Ill.
Armstrong, Baxter Hardin, 3801 Concord Blvd., Concord, Calif.
Bird, Joseph Francis (*J*), 708 Moosic St., Scranton, Pa.
Blake, Arthur E. (*J*), 5 Vets A and M Trailer Camp, Ft. Collins, Colo.
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Cahill, Rev. Anthony Edmund, O.S.B., Belmont Abbey College, Belmont, N. C.
Casey, Robert George (*J*), 229 W. Winona Ave., Norwood, Pa.
Chasteen, Joseph Wiley, Jr., U.S.S. Fulton, AS-11-Div. 8, c/o Fleet Post Office, New York, N. Y.
Clark, Clifton Bob, Electrical Engineering Department, U. S. Naval Academy, Annapolis, Md.
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"Fact," as I intend the term, can only be defined ostensively. Everything that there is in the world I call a "fact." The sun is a fact; Caesar's crossing of the Rubicon was a fact; if I have toothache, my toothache is a fact. If I make a statement, my making it is a fact, and if it is true there is a further fact in virtue of which it is true, but not if it is false. The butcher says: "I'm sold out, and that's a fact"; immediately afterwards, a favored customer arrives, and gets a nice piece of lamb from under the counter. So the butcher told two lies, one in saying he was sold out, and the other in saying that his being sold out was a fact. Facts are what make statements true or false. I should like to confine the word "fact" to the minimum of what must be known in order that the truth or falsehood of any statement may follow analytically from those asserting that minimum. For example, if "Brutus was a Roman" and "Cassius was a Roman" each assert a fact, I should not say that "Brutus and Cassius were Romans" asserted a new fact. We have seen that the questions whether there are negative facts and general facts raise difficulties. These niceties, however, are largely linguistic.—Bertrand Russell, Human Knowledge 1948.